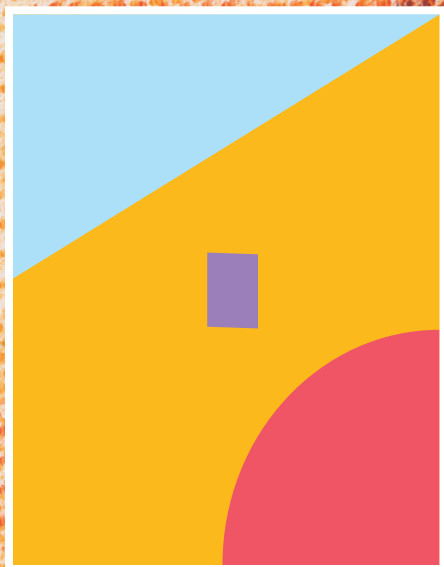


Models in Microeconomic Theory

Expanded Second Edition

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6 Producer behavior

We suggest that you begin by responding to the following question, to which we return at the end of the chapter.

Assume that you are a vice president of a package delivery company. The company employs 196 workers in addition to its management team. It was founded five years ago and is owned by three families.

The work is unskilled; each worker needs one week of training. All the company's employees have been with the company for three to five years. The company pays its workers more than the minimum wage and provides the benefits required by law. Until recently, it was making a large profit. As a result of a recession, profit has fallen significantly, but is still positive.

You attend a meeting of management to decide whether to lay off some workers. The company's Finance Department has prepared the following forecast of annual profit.

Number of workers who will continue to be employed	Expected annual profit in millions of dollars
0 (all workers will be laid off)	Loss of 8
50 (146 workers will be laid off)	Profit of 1
65 (131 workers will be laid off)	Profit of 1.5
100 (96 workers will be laid off)	Profit of 2
144 (52 workers will be laid off)	Profit of 1.6
170 (26 workers will be laid off)	Profit of 1
196 (no layoffs)	Profit of 0.4

How many workers would you continue to employ?

6.1 The producer

Producers, like consumers, play a central role in economic models. A consumer can trade goods, changing the distribution of goods among the agents in the economy. A producer can change the availability of goods, transforming inputs, which may be physical goods, like raw materials, or mental resources, like information and attention, into outputs.

In the model we study, a producer is specified by (i) a technology, which describes her ability to transform inputs into outputs, and (ii) the motives that guide her decision regarding the amounts of inputs and outputs. Many producers are not individuals, but organizations, like collectives, families, or firms. Such organizations typically have hierarchical structures and mechanisms to make collective decisions. The model we study does not consider how these mechanisms affect the production decision.

We study a simple model in which the producer can transform a single good, input, into another good, output. The technology available to her is modeled by a function f , where $f(a) = y$ means that the quantity a of input yields y units of output. We assume that a positive output requires a positive input ($f(0) = 0$) and more input produces at least as much output (f is nondecreasing). We further assume that the impact on output of an extra unit of input is no larger for large amounts of the input than it is for small amounts of the input (f is concave) and this impact goes to zero as the amount of input increases without bound (the producer's effectiveness is spread more thinly as output increases).

Definition 6.1: Production function

A *production function* is a function $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, giving the quantity of output for any quantity of the input, that is continuous, nondecreasing, and concave, satisfies $f(0) = 0$, and has the property that for any $\varepsilon > 0$ there is a quantity y such that $f(y + 1) - f(y) < \varepsilon$.

We assume that the producer operates in an environment in which she has to pay for the input and is paid for the output according to given prices. In this chapter we assume that the producer believes her actions do not affect these prices. That is, the producer, like the consumer in the previous chapters, is a price-taker. This assumption fits situations in which the amount of input used by the producer and the amount of her output are both small compared with the total amounts of these goods in the economy, so that the producer's actions have little effect on the aggregates and thus do not significantly affect the prices of the goods. The assumption is not appropriate for a producer whose use of an input or production of an output dominates the markets for those goods, so that her actions do affect the market prices. We consider such a producer in the next chapter.

Denote the price of output by p and the price of input by w . A producer who uses a units of input to produce y units of output obtains the revenue py and profit $\pi = py - wa$. We assume that the producer has preferences over the triples (a, y, π) . That is, she potentially cares about the amount of input she uses, the amount of output she produces, and the amount of profit she obtains. Here are some possible forms for her preferences.

Output maximization

The producer prefers (a, y, π) to (a', y', π') if (i) $y > y'$ and $\pi \geq 0$, or (ii) $\pi \geq 0$ and $\pi' < 0$. Such a producer chooses (a, y) to maximize output subject to profit being nonnegative. The producer's preference for profit to be nonnegative may be due to the difficulty of surviving when she makes a loss.

Profit maximization

The producer prefers (a, y, π) to (a', y', π') if and only if $\pi > \pi'$. Such a producer chooses (a, y) to maximize profit.

Profit maximization with lower bar on output

For some given output \bar{y} , the producer prefers (a, y, π) to (a', y', π') if (i) $\pi > \pi'$ and $y \geq \bar{y}$, or (ii) $y \geq \bar{y}$ and $y' < \bar{y}$. Such a producer chooses (a, y) to maximize profit subject to producing at least \bar{y} . If \bar{y} is small, the constraint does not bind. But if it is large, it constrains the amount of the input to at least the number \bar{a} for which $f(\bar{a}) = \bar{y}$.

A cooperative

Assume that the producer is an organization that decides the number of its members and divides its profit equally among them. It employs only its own members. Each member contributes one unit of labor, so that the amount a of the input is the number of members of the cooperative. The cooperative aims to maximize the profit per member, π/a .

In this chapter we explore the implications of only the first two forms of preferences: output maximization and profit maximization.

6.2 Output maximization

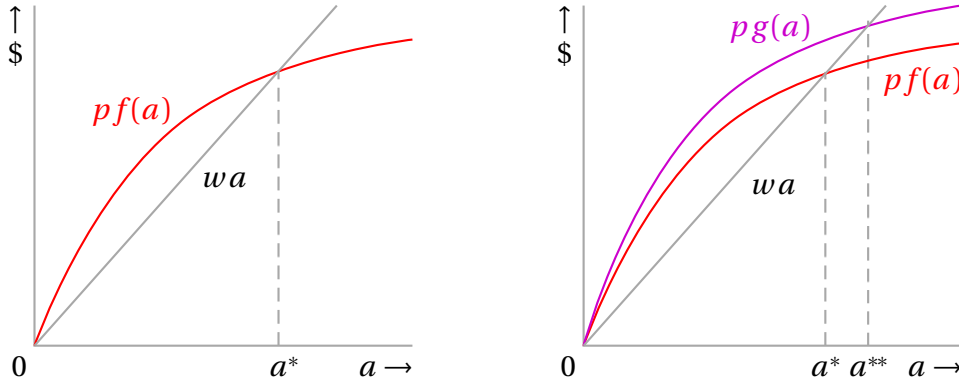
We start by considering a producer who aims to maximize the amount of output subject to not making a loss.

Definition 6.2: Output-maximizing producer

Given the prices p for output and w for input, an *output-maximizing producer* with *production function* f chooses the amount a of input to solve the problem

$$\max_a f(a) \text{ subject to } pf(a) - wa \geq 0.$$

Figure 6.1a illustrates such a producer's decision problem. The producer's profit is given by the difference between the red curve labelled $pf(a)$ and the line labelled wa . If, in addition to the assumptions we have made about the production function f , it is strictly concave, then either profit is negative for all



(a) A case in which an output-maximizing producer chooses a positive amount, a^* , of the input.

(b) The effect of a technological improvement for an output-maximizing producer.

Figure 6.1 An output-maximizing producer

positive values of a or there is a unique positive number a^* such that $pf(a^*) - wa^* = 0$. (Figure 6.1a illustrates the second case.)

Proposition 6.1: Optimal input for output-maximizing producer

If the **production function** f is strictly concave then the amount of input chosen by an **output-maximizing producer with production function** f facing the price w of input and the price p of output is

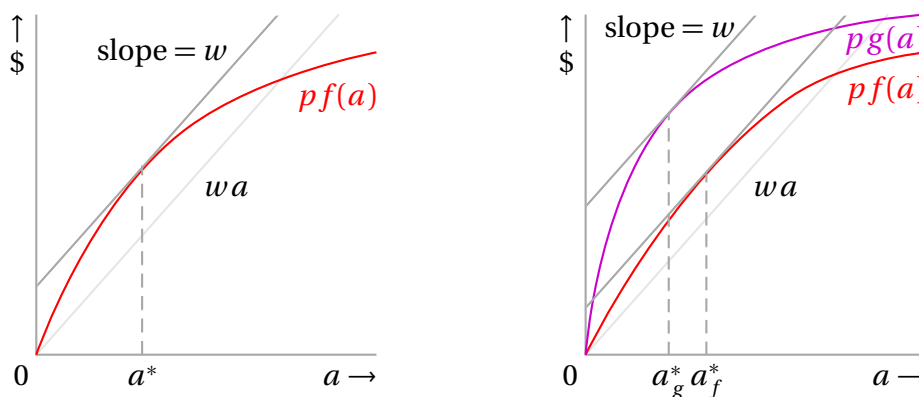
$$\begin{cases} 0 & \text{if } pf(a) - wa < 0 \text{ for all } a > 0 \\ a^* & \text{otherwise} \end{cases}$$

where a^* is the unique positive number for which $pf(a^*) - wa^* = 0$.

The implications for the producer's optimal action of changes in the prices of the input and output and in the technology follow immediately from Figure 6.1a (for prices) and from Figure 6.1b (for technology).

Proposition 6.2: Comparative statics for output-maximizing producer

If the **production function** f is strictly concave then a decrease in the price of the input, an increase in the price of output, and a technological improvement that changes the production function from f to g with $g(a) \geq f(a)$ for all a , all cause the amount of input (and output) chosen by an **output-maximizing producer with production function** f to increase or stay the same.



(a) The amount a^* of input chosen by a profit-maximizing producer.

(b) A possible effect of an improvement in technology for a profit-maximizing producer.

Figure 6.2 A profit-maximizing producer

6.3 Profit maximization

Producers are more commonly assumed to be profit-maximizers than output-maximizers.

Definition 6.3: Profit-maximizing producer

Given the prices p for output and w for input, a *profit-maximizing producer* with *production function* f chooses the amount of input to solve the problem

$$\max_a pf(a) - wa.$$

Figure 6.2a illustrates such a producer's decision problem. If the production function is differentiable and strictly concave then a solution of the producer's problem is characterized as follows.

Proposition 6.3: Optimal input for profit-maximizing producer

If the *production function* f is differentiable and strictly concave then the amount of input chosen by a *profit-maximizing producer* with *production function* f facing the price w of input and the price p of output is

$$\begin{cases} 0 & \text{if } pf(a) - wa < 0 \text{ for all } a > 0 \\ a^* & \text{otherwise} \end{cases}$$

where a^* is the unique positive number for which $pf'(a^*) - w = 0$.

Proof

The producer's profit when she chooses the amount a of input is $pf(a) - wa$. Given that f is strictly concave, this function is strictly concave in a . The result follows from the standard conditions for a maximizer of a differentiable function.

A change in the price of input or the price of output changes the amount of input chosen by a profit-maximizing producer in the same direction as it does for an output-maximizing producer.

Proposition 6.4: Comparative statics for profit-maximizing producer

An increase in the price of input or a decrease in the price of output causes the amount of input chosen by a **profit-maximizing producer** to decrease or remain the same.

Proof

Denote by $\alpha(w)$ the amount of input chosen by the producer when the input price is w . By definition,

$$pf(\alpha(w)) - w\alpha(w) \geq pf(a) - wa \text{ for all } a,$$

or

$$p[f(\alpha(w)) - f(a)] \geq w[\alpha(w) - a] \text{ for all } a.$$

In particular, for the prices w^1 and w^2 of the input,

$$p[f(\alpha(w^1)) - f(\alpha(w^2))] \geq w^1[\alpha(w^1) - \alpha(w^2)]$$

and

$$p[f(\alpha(w^2)) - f(\alpha(w^1))] \geq w^2[\alpha(w^2) - \alpha(w^1)].$$

Adding these inequalities yields

$$0 \geq (w^1 - w^2)(\alpha(w^1) - \alpha(w^2)).$$

Thus if $w^1 < w^2$ then $\alpha(w^1) \geq \alpha(w^2)$. A similar argument applies to changes in the price p of output.

Note that this proof does not use any property of the production function, so that the result in particular does not depend on the concavity of this function.

If the production function is differentiable, we can alternatively prove the result as follows, given Proposition 6.3. If $p f'(0) < w$ then increasing w preserves the inequality and the optimal production remains 0. If $p f'(0) \geq w$ then increasing w does not increase the solution of the equation $p f'(a) = w$, given that f is concave.

Unlike an output-maximizer, a profit-maximizer may decrease output when the technology improves; Figure 6.2b gives an example. However, as you can verify, if the production function is differentiable and the technological improvement from f to g is such that $g'(a) \geq f'(a)$ for all a , then a profit-maximizing producer does increase the amounts of input and output.

6.4 Cost function

Given a production function, we can find the cost of producing any amount of output. Specifically, for the production function f , the cost of producing y units of output is $w f^{-1}(y)$. Sometimes it is convenient to take the cost function as the primitive of the model, rather than deriving it from the production function. That is, we start with a function C that specifies the cost $C(y)$ of producing any amount y of output. This approach is appropriate if we are interested only in the market for output.

A natural assumption is $C(0) = 0$. We assume also that the average cost $C(y)/y$ of producing y units eventually exceeds any given bound. Some cost functions C have the form $C(y) = k + c(y)$ for $y > 0$, where $k > 0$ and c is an increasing function with $c(0) = 0$. In such cases, we refer to k as the fixed cost of production and to $c(y)$ as the variable cost.

Definition 6.4: Cost function

A *cost function* is an increasing function $C : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $C(0) = 0$ such that for all $L > 0$ there exists (a large) number y such that $C(y)/y > L$. If this function takes the form

$$C(y) = \begin{cases} 0 & \text{if } y = 0 \\ k + c(y) & \text{if } y > 0 \end{cases}$$

for some increasing function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $c(0) = 0$ and some $k > 0$, then C is called a *cost function with a fixed cost k and variable cost function c* .

The function $AC(y) = C(y)/y$ for $y > 0$ is the *average cost function* for C , and if C is differentiable at y then $MC(y) = C'(y)$ is the *marginal cost* for C at y .

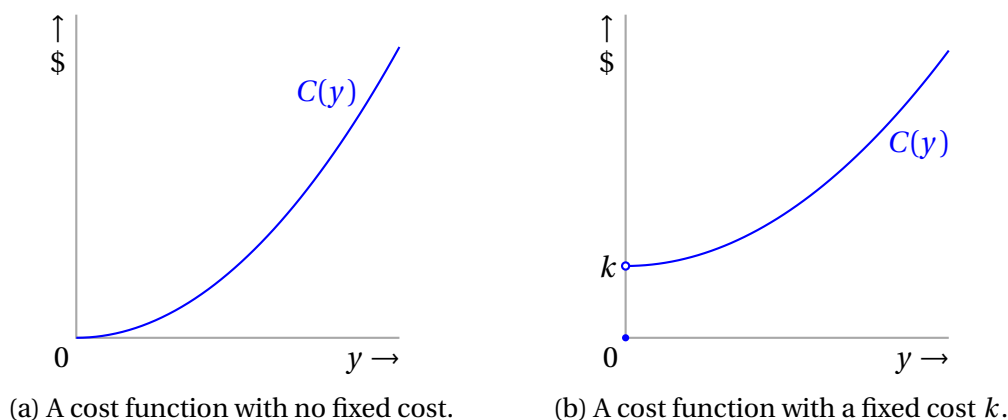


Figure 6.3

Cost functions with and without a fixed cost are shown in Figures 6.3a and 6.3b. Average and marginal cost functions are shown in Figures 6.4a and 6.4b.

Note that if output is produced by a single input, as we assumed before, and the production function is f , then $C(y) = w f^{-1}(y)$, a cost function with no fixed cost (C is increasing since f is increasing). Given that f is concave, C is convex in this case.

The following properties of the average and marginal cost functions are sometimes useful.

Proposition 6.5: Properties of the average and marginal cost functions

Let C be a **cost function** (without any fixed cost).

- If C is convex then AC is nondecreasing.
- If C is differentiable then $\lim_{y \rightarrow 0} AC(y) = MC(0)$.
- If C is differentiable then AC is increasing at y if $AC(y) < MC(y)$ and decreasing at y if $MC(y) < AC(y)$.

Proof

- Let $a > b$ and $\lambda = b/a$. Then $\lambda a + (1 - \lambda)0 = b$, so that by the convexity of C we have $C(b) \leq \lambda C(a) + (1 - \lambda)C(0) = \lambda C(a)$ and thus $AC(b) = C(b)/b \leq C(a)/a = AC(a)$.

- We have

$$\lim_{y \rightarrow 0} AC(y) = \lim_{y \rightarrow 0} \frac{C(y)}{y} = \lim_{y \rightarrow 0} \frac{C(y) - C(0)}{y - 0} = C'(0).$$

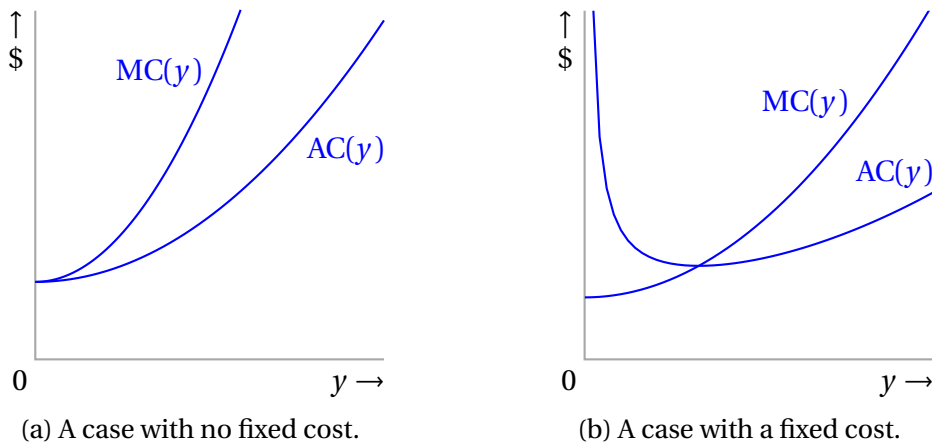


Figure 6.4 Average and marginal cost functions.

c. Differentiating AC we get

$$AC'(y) = C'(y)/y - C(y)/y^2 = (MC(y) - AC(y))/y,$$

from which the result follows.

An intuition for part c of the result is that $C(y)$ is the sum of the marginal costs up to y , so that $AC(y)$ is the average of $MC(z)$ for $0 \leq z \leq y$. Thus if $MC(y) > AC(y)$ and y increases then we add a cost greater than $AC(y)$, so that the average increases.

The profit of a producer with cost function C who faces the price p for output and produces y units of output is

$$py - C(y).$$

The following result, for an output-maximizing producer, is immediate. An example in which the producer's optimal output is positive is given in [Figure 6.5a](#).

Proposition 6.6: Output chosen by output-maximizing producer

An **output-maximizing producer** with **cost function** C who faces the price p for output chooses the largest positive output y^* for which $C(y^*)/y^* = AC(y^*) = p$ if such an output exists, and otherwise chooses output 0.

The output chosen by a profit-maximizing producer with a convex differentiable cost function is also easy to characterize. An example in which the producer's optimal output is positive is shown in [Figure 6.5b](#).

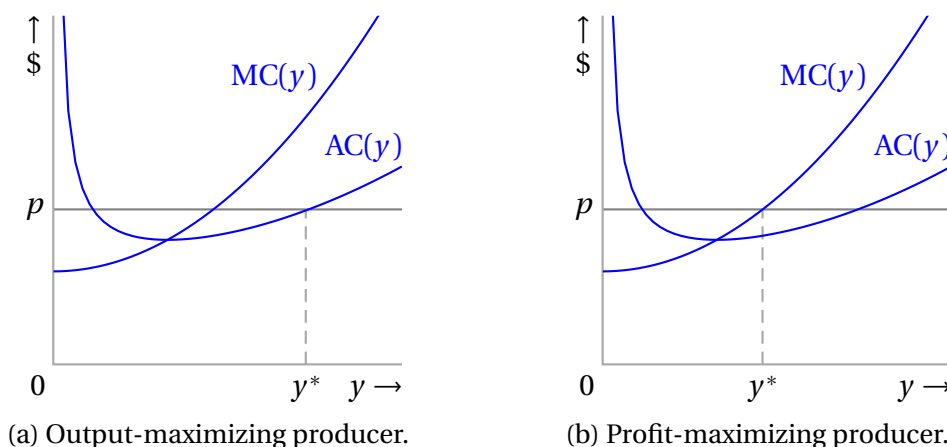


Figure 6.5 The output chosen by a producer facing the price p .

Proposition 6.7: Output chosen by profit-maximizing producer

A **profit-maximizing producer** with a convex differentiable **cost function** C who faces the output price p chooses an output y^* for which $C'(y^*) = MC(y^*) = p$ if $p \geq AC(y^*)$ and otherwise chooses output 0.

Proof

Given the convexity of C , the function $py - C(y)$ is concave in y , so that the result follows from the standard conditions for a maximizer of a differentiable function.

6.5 Producers' preferences

We have discussed two possible preferences for producers, output maximization and profit maximization. Many other textbooks restrict attention to profit maximization. By contrast, the preferences of individuals in consumer theory are usually taken to be subject only to mild assumptions (discussed in [Chapter 4](#)).

Why is profit maximization usually assumed? Some people think of it as a normative assumption: producers should maximize profit. Others consider it descriptive: the main goal of producers is to maximize profit. Some researchers think that the assumption of profit-maximization is made only because it allows economists to draw analytically interesting and nontrivial results. Yet others think that the assumption is so that students believe that profit-maximization is the only legitimate goal of producers.

We refrain from expressing our opinion on the issue. We suggest only that you consider it taking into account how students of economics and other disciplines have responded to the question at the beginning of the chapter.

Did you decide to maximize profit and lay off 96 workers? Or did you decide to give up all profit and not lay off any worker? Or did you compromise and choose to lay off only 26 or 52 workers? Surely you did not lay off more than 96 workers, since doing so is worse than laying off 96 in terms of both the number of layoffs and profit.

When the question is posed to students in various disciplines, students of economics tend to lay off more workers than students in philosophy, law, mathematics, and business. It is not clear whether this effect is due to selection bias (students who choose to study economics are different from students in the other disciplines) or to indoctrination (studying material in which profit-maximization is assumed has an effect). In any case, even among students of economics, only about half choose the profit-maximizing option. So maybe profit-maximization is not the only goal of producers that we should investigate?

Problems

1. *Comparative statics.* Propositions 6.2 and 6.4 give comparative static results for a producer with a concave production function. Consider analogous results for a producer with a convex cost function.

For an [output maximizer](#) and a [profit maximizer](#), analyze diagrammatically the effect of (a) an increase in the price of output and (b) a technological change such that all marginal costs decrease.

2. *Two factories.* A producer can use two factories to produce output. The production functions for the factories are $f(a_1) = \sqrt{a_1}$ and $g(a_2) = \sqrt{a_2}$, where a_i is the amount of input used in factory i . The cost of a unit of input is 1 and the cost of activating a factory is $k > 0$. Calculate the producer's cost function.
3. *A producer with a cost of firing workers.* A producer uses one input, workers, to produce output according to a production function f . She has already hired a_0 workers. She can fire some or all of them, or hire more workers. The wage of a worker is w and the price of output is p . Compare the producer's behavior if she maximizes profit to her behavior if she also takes into account that firing workers causes her to feel as if she bears the cost $l > 0$ per fired worker.

4. *Robinson Crusoe*. Robinson Crusoe is both a producer and a consumer. She has one unit of time, which she can divide between leisure and work. If she devotes the amount of time x to work then her output is $f(x)$, where f is increasing and strictly concave. She has a [monotone convex preference relation](#) over the set of leisure–consumption pairs (l, c) that is represented by a differentiable [utility function](#) u .
- Formally state the problem that Crusoe's optimal choice of (l, c) solves.
 - Calculate the solution of Crusoe's problem for $f(x) = \sqrt{x}$ and $u(l, c) = lc$.
 - Explain why the [marginal rate of substitution](#) between leisure and consumption at the pair (l^*, c^*) chosen by Crusoe is equal to $f'(1 - l^*)$ (the marginal product at $1 - l^*$).

(More difficult) Now assume that Crusoe has two independent decision-making units. One unit decides the amount to produce and the other decides how much to consume. The units make their decisions simultaneously. Each unit takes the value of a unit of time devoted to work to be some number w . The consumption unit chooses a leisure-consumption pair (l, c) that maximizes $u(l, c)$ given the budget constraint $c = w(1 - l) + \pi$, where π is the profit of the production unit. The production unit maximizes its profit given that it has to pay w for a unit of time devoted to work.

The units are in harmony in the sense that given the price w , the decision of how much to consume is consistent with the decision of how much to produce.

- Give the formal conditions required for harmony between the consumption and production units to prevail.
- Find the value of w^* , and the associated pair (l^*, c^*) , that satisfies the conditions in the previous part when $f(x) = \sqrt{x}$ and $u(l, c) = lc$.
- Demonstrate graphically why Crusoe behaves in the same way if she makes her decision as in the first part of the problem as she does if she makes her decision using two separate units, as in the second part of the problem.
- Suppose that Crusoe's production unit acts as an output maximizer rather than a profit maximizer. Show diagrammatically that the pair (l^*, c^*) that is in harmony differs from the pair that maximizes $u(l, c)$ subject to $c = f(1 - l)$.

Notes

See [Rubinstein \(2006b\)](#) for the issue discussed in [Section 6.5](#). The exposition of the chapter draws upon [Rubinstein \(2006a, Lecture 6b\)](#).

