Models in Microeconomic Theory

Expanded Second Edition

Martin J. Osborne Ariel Rubinstein





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7 Monopoly

7.1 Basic model

In the previous chapter we consider a producer who acts as if her behavior has no effect on the prices of the input or output. We argue that this assumption may be appropriate if the producer's quantities of inputs and output are small relative to the total volume of trade in the markets.

In this chapter we study several variants of a model that fits a very different situation, in which the producer of a single good is the only one serving a market. The variants differ in the type of options the producer can offer potential buyers. In the basic case, the producer can post a price per unit, and each buyer can purchase any amount of the good at that price. In other cases, the producer has other instruments like offering all consumers a set of price-quantity pairs. In each case, every potential buyer chooses the option she most prefers. The producer predicts correctly the buyers' responses and acts to advance her target (like maximizing profit or increasing production).

We allow for the possibility that the market has a number of segments, with distinct demand functions. Thus a specification of the market consists of two elements, (*i*) a demand function for each segment and (*ii*) a description of a producer, which includes her cost function and preferences.

Definition 7.1: Monopolistic market

A *monopolistic market* $\langle (q_i)_{i=1}^k, C, \succeq \rangle$ for a single good has the following components.

Demand

A collection $(q_i)_{i=1}^k$ of decreasing functions, where $q_i : \mathbb{R}_+ \to \mathbb{R}_+$. The function q_i , the *demand function in segment i*, associates with every price p_i for segment *i* the total amount $q_i(p_i)$ of the good demanded in that segment.

Producer

A single producer, called a *monopolist*, characterized by a cost function C that is continuous and convex and satisfies C(0) = 0, and a preference

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relation \succeq over pairs $((y_1, ..., y_k), \pi)$, where y_i is the quantity sold in segment *i* for i = 1, ..., k and π is the producer's profit.

7.2 Uniform-price monopolistic market

We first consider a monopolist who sets a single price, the same for all segments of the market. The monopolist might act in this way because she is prohibited by law from setting different prices for different segments of the market. (For example, charging men and women different prices may be outlawed.) Also, a producer's ability to enforce different prices in different segments of the market is limited if individuals can buy the good in one segment at a low price and sell it in another segment at a high price. (Such arbitrage is easier for some goods, like books, than it is for others, like haircuts.)

Definition 7.2: Uniform-price monopolistic market

A *uniform-price monopolistic market* is a monopolistic market in which the producer chooses a single price, the same in all segments.

7.2.1 Profit-maximizing monopolist

Let $\langle (q_i)_{i=1}^k, C, \succeq \rangle$ be a uniform-price monopolistic market. Define the total demand function Q by $Q(p) = \sum_{i=1}^k q_i(p)$ for all p. The profit of a producer who sets the price p in a uniform-price monopolistic market is $\pi(p) = pQ(p) - C(Q(p))$. Given that each function q_i is decreasing, the function Q is decreasing, and hence has an inverse, say P. Thus the producer's setting a price p and obtaining the profit $\pi(p)$ is equivalent to her choosing the output y = Q(p) and obtaining the profit $\Pi(y) = P(y)y - C(y)$.

A useful concept in the analysis of a uniform-price monopolistic market is marginal revenue.

Definition 7.3: Marginal revenue

The *marginal revenue* at the output *y* for the differentiable (demand) function *Q* is

$$MR(y) = [P(y)y]' = P(y) + P'(y)y,$$

where *P* is the inverse of *Q*.

The number MR(y) is the rate of change in revenue as output increases. If the function *P* is decreasing, we have $MR(y) \le P(y)$ for all *y*. The intuitive reason for

this inequality is that selling an additional unit of the good increases revenue by approximately P(y) but also causes a reduction in the price of all y units sold.

Usually the function MR is assumed to be decreasing, but this property does not follow from the assumptions we have made. The derivative of MR at y is 2P'(y) + P''(y)y, so that if P''(y) is positive and large enough, the derivative is positive even if P is decreasing. The following example illustrates this point in an environment in which the good is indivisible.

Example 7.1: Monopoly with marginal revenue that is not decreasing

Consider a market for an indivisible good and three consumers, each of whom buys either one unit of the good or no units. One consumer buys one unit of the good if and only if the price is at most 10; the cutoff prices for the two other consumers to buy a unit are 6 and 5. In this context, P(y) is the highest price at which the producer can sell y units of the good. Thus P(1) = 10, P(2) = 6, and P(3) = 5. Given the indivisibility of the good, we define the marginal revenue at the output y as MR(y) = P(y)y - P(y-1)(y-1), yielding the following numbers, where MR(3) > MR(2).

у	Revenue(y)	MR(y)
1	10	10
2	12	2
3	15	3

The next result gives a necessary condition for an output to maximize the profit of a producer in a uniform-price monopolistic market.

Proposition 7.1: Uniform-price profit-maximizing monopolist

Consider a uniform-price monopolistic market $\langle (q_i)_{i=1}^k, C, \succeq \rangle$ in which *C* and each function q_i is differentiable. For any price *p*, let $Q(p) = \sum_{i=1}^k q_i(p)$, let MR be the marginal revenue function for *Q*, and let MC be the marginal cost function for *C*. If the monopolist's preferences \succeq are profit-maximizing and her optimal output y^* is positive, then MR(y^*) = MC(y^*).

Proof

The monopolist chooses p to maximize pQ(p) - C(Q(p)), or equivalently y to maximize P(y)y - C(y), where P is the inverse of Q. The result follows from the standard necessary condition for an interior maximizer of a differentiable function.





(a) A case in which MR(y) = MC(y) for only one value of *y*.

(b) A case in which MR(y) = MC(y) for multiple values of *y*.

Figure 7.1 The output y^* chosen by a profit-maximizing producer in a uniform-price monopolistic market, and the resulting price $P(y^*)$.

Note that the condition in the result is only necessary, not sufficient. In the left panel of Figure 7.1, a single output satisfies the condition, and this output maximizes profit. In the right panel, three outputs satisfy the condition. The output labeled y^* is the profit-maximizer, because the difference between the area under the MR curve (the total revenue) and the area under the MC curve (the total cost) up to y^* is larger than the difference between these areas for y_1 and y_2 .

Inefficiency Since MR(y) < P(y) for all y, an implication of Proposition 7.1 is that the price charged by a profit-maximizing producer in a uniform-price monopolistic market is greater than the marginal cost at this output. As a consequence, an inefficiency of sorts exists in such a market: the cost of production of another unit of the good is less than the price that some buyers are willing to pay for the good. The monopolist does not produce the extra unit because she takes into account that the price reduction necessary to sell the extra unit will affect all the other units, too, causing her profit to fall.

Sometimes the area under the inverse demand function,

$$W(y) = \int_{0}^{y} P(x) \, dx,$$
(7.1)

is used as a measure of the consumers' welfare when y units of the good are sold. The logic behind this definition is clear when the good is indivisible, each consumer either buying one unit or none. The number P(1) is then the highest price that any consumer in the market is willing to pay for the good, P(2) is the



Figure 7.2 The reduction in W(y) caused by the reduction in output from y_c to y^* .

highest price any remaining consumer is willing to pay, and so forth. The integral is analogous to the sum $P(1) + P(2) + \cdots + P(y)$, the maximum amount of money for which *y* units can be sold.

The area under the marginal cost function between 0 and *y* is the total cost of producing *y*. Thus it is common to interpret the integral between 0 and *y* of the difference between the demand function and the marginal cost function as a measure of the welfare added to the world by the production of *y* units of the good. This measure of welfare is maximized at the quantity y_c for which $P(y_c) = MC(y_c)$. Thus the loss of welfare due to the operation of the producer as a monopolist is the yellow triangle in Figure 7.2. This triangle is called the *deadweight loss* due to the monopoly.

Two policies to control a monopolist's behavior involve setting a maximum price and providing the monopolist with a subsidy.

Maximum price If the maximum price the monopolist is allowed to charge is p_{max} , then for outputs y with $P(y) < p_{\text{max}}$ the value of MR remains the same as before, while for outputs such that $p(y) > p_{\text{max}}$ we have $MR(y) = p_{\text{max}}$. That is, the function MR is not continuous and has two segments, as shown in Figure 7.3a.

If p_{max} is set equal to the price $P(y_c)$, where y_c is the output for which $P(y_c) = MC(y_c)$, then the producer chooses y_c , reducing her profit and eliminating the deadweight loss.

Subsidy Suppose that the producer gets a subsidy of t units of money for each unit she sells, in addition to the amount the consumers pay. Such a subsidy raises the MR curve by t, so that the intersection of the new MR and MC is at a higher quantity. For an appropriate value of the subsidy the monopolist optimally produces the quantity y_c (see Figure 7.3b). However, if the consumers pay the subsidy then this policy may not improve their welfare.





(a) The output chosen by a monopolist who can set a price of at most p_{max} .

(b) The output chosen by a monopolist who receives a subsidy of *t* per unit sold.

Figure 7.3 The effect of policies to change the output of a producer in a uniform-price monopolistic market.

7.2.2 Output-maximizing monopolist

As we discussed in the previous chapter, profit maximization is not the only possible target for a producer. Consider a monopolist who maximizes output subject to obtaining nonnegative profit. Such a monopolist produces the quantity y^* for which $AC(y^*) = P(y^*)$ (see Figure 7.4). This output is larger than the output y_c that maximizes the consumers' welfare W(y).

7.3 Discriminatory monopoly

We now consider a monopolistic market in which the producer can set different prices in different segments.

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Definition 7.4: Discriminatory monopolistic market
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A *discriminatory monopolistic market* is a monopolistic market in which the producer chooses a collection of prices, one for each segment of the market.

Note that the model assumes that the demand in each segment depends only on the price in that segment. In particular, this demand does not depend on the prices in other segments, so that we are assuming implicitly that consumers' demands are not affected by any feeling they may have that charging different prices to different groups is unfair.



Figure 7.4 The output y^* chosen by an output-maximizing producer in a uniform-price monopolistic market, and the resulting price $P(y^*)$.

Let $\langle (q_i)_{i=1}^k, C, \succeq \rangle$ be a discriminatory monopolistic market in which the producer's preferences are profit-maximizing, so that her problem is

$$\max_{y_1,...,y_k} \left[\sum_{i=1}^k P_i(y_i) y_i - C\left(\sum_{i=1}^k y_i\right) \right],$$
(7.2)

where P_i is the inverse of q_i . Note that this problem cannot be decomposed into k independent problems because the cost is a function of the total output.

The next result generalizes the necessary condition for an output to maximize the producer's profit in a uniform-price monopolistic market (Proposition 7.1).

Proposition 7.2: Discriminatory profit-maximizing monopolist

Consider a discriminatory monopolistic market $\langle (q_i)_{i=1}^k, C, \succeq \rangle$ in which *C* and each function q_i is differentiable. For each segment *i*, let MR_i be the marginal revenue function for q_i , and let MC be the marginal cost function for *C*. If the monopolist's preferences are profit-maximizing and her optimal output y_i^* in segment *i* is positive, then

$$\mathrm{MR}_{i}(y_{i}^{*}) = \mathrm{MC}\left(\sum_{j=1}^{k} y_{j}^{*}\right).$$

Proof

The result follows from the standard necessary condition for a maximizer of a differentiable function, applied to (7.2).



Figure 7.5 The outputs chosen by a profit-maximizing producer in each segment of a discriminatory monopolistic market. In segment *i* the output is y_i^* and the price is $P_i(y_i^*)$.

Two intuitions lie behind this result. First, the marginal revenues for all segments in which output is positive must be the same since otherwise the producer could increase her profit by moving some production from a segment with a low marginal revenue to one with a high marginal revenue. Second, if the marginal cost is higher than the common marginal revenue then the producer can increase her profit by reducing production, and if the marginal cost is smaller than the common marginal revenue she can increase her profit by increasing production.

The result is illustrated in Figure 7.5. The curve MR(y) is the horizontal sum of the MR_{*i*} curves. For any output *y*, MR(*y*) is the marginal revenue of the monopolist given that she allocates the output *y* optimally between the segments.

7.4 Implicit discrimination

In this section we assume that the monopolist is aware that the consumers have different demand functions, but cannot discriminate between them explicitly, either because she is prohibited from doing so or because she does not know who is who. We consider the possibility that she can offer an arbitrary set of pairs (q, m), where q is an amount of the good and m is the (total) price of purchasing q. She offers the same set to all consumers, each of whom is limited to choosing one member of the set or not buying the good at all.

Specifically, we consider a market for a good that can be consumed in any quantity between 0 and 1. Each consumer *i* is willing to pay up to $V^i(q)$ for the quantity *q*, where the function V^i is increasing and continuous, and $V^i(0) = 0$. A single producer (a monopolist) produces the good at no cost.

The monopolist offers a finite set of pairs (q, m), referred to as a menu. If consumer *i* chooses (q, m), then her utility is $V^i(q) - m$. Each consumer chooses



Figure 7.6 An example of a monopolistic market with a menu in which the profit-maximizing menu for the monopolist contains two options, $(q^*, V^2(q^*))$ and $(1, m_1^*)$, where $m_1^* = V^1(1) - (V^1(q^*) - V^2(q^*))$.

an option in the menu for which her utility is highest, if this maximal utility is nonnegative; otherwise, she buys nothing. The monopolist assumes that the consumers behave optimally and chooses a menu for which her profit, the total amount paid by the consumers, is maximal.

Definition 7.5: Monopolistic market with a menu

A monopolistic market with a menu has the following components.

Demand

A collection $(V^i)_{i=1}^n$ of increasing continuous functions, where V^i : [0,1] $\rightarrow \mathbb{R}_+$ and $V^i(0) = 0$. The function V^i is the *value function for consumer i*, giving the maximum amount $V^i(q)$ consumer *i* is willing to pay for *q* units of the good.

Producer

A single producer, called a *monopolist*, with no costs, who chooses a set M of pairs, called a *menu*, where a pair (q, m) represents the option to buy q units of the good at the (total) price m.

If $V^i(q) - m \ge 0$ for some $(q, m) \in M$, consumer *i* chooses an option $(q, m) \in M$ for which $V^i(q) - m$ is maximal; otherwise she buys nothing. The producer chooses *M* so that the consumers' choices maximize her profit.

We now analyze a monopolistic market with a menu in which there are two consumers, one of whom values each additional unit of the good more than the other. One implication of the following result is that for some such markets, offering a menu that consists of more than one pair is optimal for the monopolist.

Proposition 7.3: Monopolistic market with a menu

Consider a two-consumer monopolistic market with a menu (V^1, V^2) in which $V^1(q^1) - V^1(q^2) > V^2(q^1) - V^2(q^2)$ whenever $q^1 > q^2 \ge 0$. Let q^* be a maximizer of $2V^2(q) - V^1(q)$. The monopolist's maximal profit is

$$\max\{V^{1}(1), 2V^{2}(1), V^{1}(1) + 2V^{2}(q^{*}) - V^{1}(q^{*})\}.$$

- If *V*¹(1) is the largest term, then {(1, *V*¹(1))} is an optimal menu and consumer 1 alone purchases the single option.
- If 2*V*²(1) is the largest term, then {(1, *V*²(1))} is an optimal menu and both consumers purchase the single option.
- If $V^1(1) + 2V^2(q^*) V^1(q^*)$ is the largest term, then $M^* = \{(q^*, V^2(q^*)), (1, V^1(1) (V^1(q^*) V^2(q^*)))\}$ is an optimal menu; consumer 2 purchases the first option and consumer 1 purchases the second option.

Proof

First note that the monopolist cannot gain by offering options not chosen by any consumer. Thus an optimal menu consisting of one or two options exists.

Consider menus that consist of a single option, (q, m). (*i*) If $V^1(q) < m$, then neither consumer chooses the option and the monopolist's profit is 0. (*ii*) If $V^2(q) < m \le V^1(q)$, then consumer 1 alone chooses the option. Out of these menus, the best one for the monopolist is $\{(1, V^1(1))\}$, which yields the profit $V^1(1)$. (*iii*) If $m \le V^2(q)$, then both consumers choose the option. Out of these menus, the best one for the monopolist is $\{(1, V^2(1))\}$, which yields the profit $2V^2(1)$.

Now consider the set \mathcal{M}_2 of menus that consist of two options, one chosen by each consumer. The menu \mathcal{M}^* specified in the proposition belongs to \mathcal{M}_2 . (Consumer 1 is indifferent between the two options. Consumer 2 is indifferent between $(q^*, V^2(q^*))$ and not buying anything, and her utility from $(1, V^1(1) - (V^1(q^*) - V^2(q^*)))$ is nonpositive by the assumption about the relation between the two value functions.)

We argue that if the menu $\{(q^1, m^1), (q^2, m^2)\}$ is optimal in \mathcal{M}_2 and consumer *i* chooses (q^i, m^i) , then the menu is M^* .

Step 1
$$V^2(q^2) = m^2$$
.

Proof. Given that both consumers purchase an option, we have $V^i(q^i) \ge V^i(q^i)$

 m^i for i = 1, 2. We now argue that $V^i(q^i) = m^i$ for some i. If $V^i(q^i) > m^i$ for both consumers, then there exists $\varepsilon > 0$ (small enough) such that increasing m^i by ε increases the monopolist's profit by 2ε . We need $V^2(q^2) = m^2$ since if $V^1(q^1) = m^1$ then, given that $V^1(q) > V^2(q)$ for all q, we have $0 = V^1(q^1) - m^1 \ge V^1(q^2) - m^2 > V^2(q^2) - m^2$, contradicting $V^2(q^2) \ge m^2$.

Step 2 $(q^1, m^1) = (1, V^1(1) - (V^1(q^2) - V^2(q^2))).$

Proof. For consumer 1 to choose (q^1, m^1) we need $V^1(q^1) - m^1 \ge V^1(q^2) - m^2 = V^1(q^2) - V^2(q^2)$ (using Step 1), or $m^1 \le V^1(q^1) - (V^1(q^2) - V^2(q^2))$. Given (q^2, m^2) , the best (q^1, m^1) satisfying this condition is $(1, V^1(1) - (V^1(q^2) - V^2(q^2))$. ⊲

By Steps 1 and 2, the optimal menu in \mathcal{M}_2 has the form { $(q^2, V^2(q^2))$, $(1, V^1(1) - (V^1(q^2) - V^2(q^2)))$ }. This menu yields the profit $V^2(q^2) + V^1(1) - (V^1(q^2) - V^2(q^2)) = 2V^2(q^2) + V^1(1) - V^1(q^2)$, which is maximized by $q^2 = q^*$. Thus M^* is optimal in \mathcal{M}_2 .

Figure 7.6 shows an example of a monopolistic market with a menu in which a menu with two options is optimal.

Problems

- 1. *Double margins*. A profit-maximizing producer in a uniform-price monopolistic market has no production cost.
 - *a*. Suppose that the good in the market is indivisible. There are two consumers, each of whom wants to purchase either one or zero units of the good. One consumer is willing to pay \$10 for the good and the other is willing to pay \$8. What price does the monopolist set?

Assume now that the monopolist does not sell the good directly to the consumers, but sells it to an intermediary, who sells it to the consumers at a uniform price.

- *b.* Under the assumptions of the previous part, find the demand function of the intermediary and analyze the behavior of the producer.
- *c*. Repeat the previous parts when the good is divisible, the monopolist's cost function is $c(y) = y^2$, and the consumer's inverse demand function is P(y) = 1 y.

- *d*. Prove that if the monopolist's cost function is convex and the consumers' inverse demand function is P(y) = A By then in the presence of an intermediary the output of the monopolist is at most her output when she sells directly to consumers.
- 2. Imposing a tax.
 - *a*. Imposing a sales tax can cause a profit-maximizing monopolist in a uniform-price monopolistic market to increase the price she charges by more than the tax. To verify this claim, consider a monopolist selling a single good who has no costs and faces two consumers, 1 and 2. Consumer *i* purchases one unit if the price she pays does not exceed v^i . Assume $v^1 = 1 > v^2 = v > 0$. Show that for some values of v and t, the imposition of a tax of t causes the price charged by the monopolist to rise by more than t.
 - *b*. Show that if the monopolist faces a linear inverse demand function P(q) = A Bq and constant marginal cost of *c*, then the price increase due to a tax of *t* is less than *t*.
- 3. *Monopolist interested in fairness*. Consider a monopolist who faces a market with two segments, with demand functions q_1 and q_2 , and has no production cost. Suppose that she maximizes profit subject to the constraint that the outcome is fair in the sense that $W_1(q_1(p_1)) p_1q_1(p_1) = W_2(q_2(p_2)) p_2q_2(p_2))$, where p_i is the price in segment *i* and $W_i(q)$ is the area under the inverse demand function P_i between 0 and q, as in (7.1). (Recall that $W_i(q)$ is a rough measure of the welfare of consumers in segment *i* from purchasing *q* units of the good.)

Formulate the optimization problem of this monopolist and solve the problem when $q_i(p_i) = a_i - p_i$ for i = 1, 2, with $a_1 \ge a_2$. Compare the outcome with the one generated by a profit-maximizing monopolist.

4. *Nonlinear prices.* Consider a market for a single indivisible good; each individual can consume either one or two units of the good. A monopolist has no cost of production and faces two consumers. Consumer i (= 1, 2) is willing to pay up to $V^i(q)$ for q units of the good, where $V^i(2) > V^i(1) > V^i(0) = 0$. The monopolist cannot discriminate between the consumers, but can offer nonlinear prices: the price of the first unit a consumer buys does not have to be the same as the price of the second unit.

Give an example in which a profit-maximizing monopolist optimally chooses a price schedule for which the price of the second unit is less than the price of the first unit, and also an example in which the reverse is true.

- 5. *Bundling*. A profit-maximizing monopolist produces two indivisible goods, *A* and *B*, at zero cost. She confronts a population in which individual *i* is willing to pay up to v_a^i for good *A*, v_b^i for good *B*, and $v_a^i + v_b^i$ for both goods. The monopolist can sell either each good separately or a bundle of both goods; she is restricted to charge the same price for all individuals. Construct two examples, one in which the monopolist's optimal policy is to offer the two goods in a bundle, and one in which the optimal policy is to sell the two goods separately.
- 6. *Two-part tariff.* Assume that a profit-maximizing monopolist with a differentiable cost function *C* confronts a single consumer, who has a continuous decreasing inverse demand function *P*. Let $W(q) = \int_0^q P(x) dx$, the maximum amount the consumer is willing to pay for *q* units of the good. The monopolist makes an offer (A, p), where *A* is the cost of the option to purchase from the monopolist and *p* is a price per unit, so that a consumer who purchases any amount x > 0 pays A + px. Formulate the monopolist's problem and prove that an output q > 0 that maximizes the monopolist's profit satisfies P(q) = MC(q).
- 7. *Implicit discrimination*. Consider a market with two consumers and a good that is available in discrete amounts. The maximum amount $V^i(q)$ that each consumer *i* is willing to pay for *q* units of the good, for q = 1 or 2, is given in the following table, which shows also each consumer's marginal valuation $MV^i(q) = V^i(q) V^i(q-1)$, the maximum amount *i* is willing to pay for an additional unit when she has q 1 units.

q	$V^1(q)$	$MV^{1}(q)$	$V^2(q)$	$MV^2(q)$
0	0		0	
1	12	12	10	10
2	19	7	13	3

Consider a profit-maximizing monopolist whose cost of production is zero.

- *a*. Find the price charged by the monopolist if she can offer only a uniform price to all consumers.
- *b*. Suppose the monopolist can offer only a single option (q, m), where q is an amount of the good and m is an amount of money. Each consumer can either pay m and get q units of the good, or buy nothing. Find the option chosen by the monopolist.
- *c*. Now suppose that the monopolist can offer the consumers a menu consisting of two such options (q, m). Each consumer either chooses one

of the options or buys nothing. Show that the menu $\{(1,10),(2,17)\}$ is profit-maximizing and yields the profit 27 (consumer 1 chooses (2,17) and consumer 2 chooses (1,10)).

- 8. *Coupons*. Some stores issue coupons, giving a discount to a customer who has one. To understand the logic of this phenomenon, consider a profitmaximizing monopolist with no production cost who faces two equal-sized groups of consumers. Each member of group 1 is willing to pay 7 for the monopolist's good and incurs a cost of 4 to search for a coupon, and each member of group 2 is willing to pay up to 5 for the good and incurs a cost of 1 to search for a coupon. What price and discount does the monopolist offer?
- 9. *Two workers and one employer*. An employer has two workers, *a* and *b*. Each worker can produce any quantity in [0, 1]. The payoff of worker i (= a, b) if she produces y_i and is paid m_i is $m_i e_i(y_i)$, where the (effort cost) function e_i is increasing, differentiable, and convex, and satisfies $e_i(0) = 0$, $e'_i(0) < 1$, and $e'_i(1) > 1$. Assume that $e'_a(y) < e'_b(y)$ for all y > 0. The employer's profit is $y_a + y_b m_a m_b$.

The employer offers a menu of contracts, each of which is a pair (y, m) with the interpretation that the employer will pay m to a worker who produces y. Each worker chooses the contract she prefers or rejects all contracts.

Show that if it is optimal for a profit-maximizing employer to offer a menu consisting of two distinct contracts, (y_a, m_a) , chosen by a, and (y_b, m_b) , chosen by b, then $e'_b(y_b) = 1$, $1 - e'_a(y_a) = e'_a(y_a) - e'_b(y_a)$, $m_a = e_a(y_a)$, and $m_b = e_b(y_b) + m_a - e_b(y_a)$.

Notes

The material in this chapter is standard. Problem 1 is based on Tirole (1988, 174).