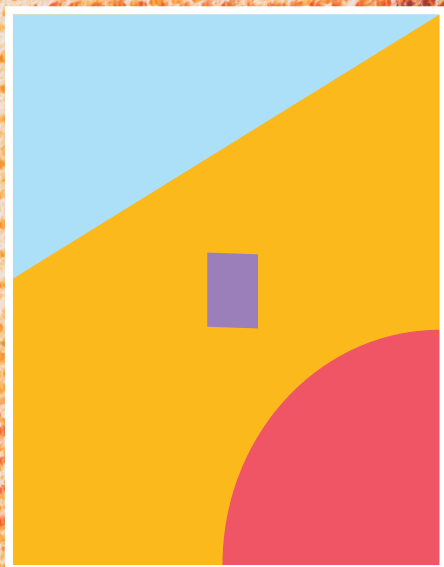


Models in Microeconomic Theory

Expanded Second Edition

Martin J. Osborne
Ariel Rubinstein





<https://www.openbookpublishers.com>

© 2020, 2023 (expanded second edition) Martin J. Osborne and Ariel Rubinstein



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs license (CC BY-NC-ND 4.0). This license allows you to share, copy, distribute, and transmit the work providing you do not modify the work, you do not use the work for commercial purposes, you attribute the work to the authors, and you provide a link to the license. Attribution should not in any way suggest that the authors endorse you or your use of the work and should include the following information.

Martin J. Osborne and Ariel Rubinstein, *Models in Microeconomic Theory* (Expanded Second Edition), Cambridge, UK: Open Book Publishers, 2023, <https://doi.org/10.11647/OBP.0361>.

To access detailed and updated information on the license, please visit <https://www.openbookpublishers.com/product/1171#copyright>.

For further details on CC BY-NC-ND licenses, see <https://creativecommons.org/licenses/by-nc-nd/4.0/>. All external links were active at the time of publication unless otherwise stated and have been archived via the Internet Archive Wayback Machine at <https://archive.org/web>.

Updated digital material and resources associated with this book are available at <https://www.openbookpublishers.com/product/1171#resources>.

Every effort has been made to identify and contact copyright holders and any omission or error will be corrected if notification is made to the publisher.

ISBN Paperback: 978-1-80511-121-4

ISBN Hardback: 978-1-80511-122-1

ISBN Digital (PDF): 978-1-80511-123-8

DOI: 10.11647/OBP0361

This book is available in two versions, one that uses feminine pronouns and one that uses masculine pronouns. This version uses feminine pronouns.

Version: 2023.5.30 (s)

13 Equilibrium with prices and expectations

In the models of markets we have discussed so far, equilibrium prices make the individuals' decisions compatible. Each individual takes the prices as given when deciding on her action, and at the equilibrium prices the demand and supply of every good are equal.

In this chapter, an individual's behavior is affected not only by the prices but also by her expectations regarding other parameters. Each individual takes these expectations, like the prices, as given. In equilibrium, each individual behaves optimally, the supply and demand for each good are equal, and the expectations of individuals are correct.

We present three models. In the first model, each individual chooses one of two bank branches. Her decision is affected only by her belief about the expected service time in each branch. In the second model, potential buyers of a used car, who cannot observe the quality of the cars for sale, take into account their expectation of the average quality of these cars as well as the price. In the third model, the unit cost of catching fish depends on the total amount of fish caught. Each fisher makes her decision taking as given both the price of fish and her expectation about the unit cost she will incur.

13.1 Distributing customers among bank branches

13.1.1 Introduction

Individuals live on the long main street of a town. At each end of the street there is a branch of a bank. Each individual cares only about the amount of time she spends dealing with the bank, which is the sum of her travel time and waiting time. The waiting time in each branch depends on the number of individuals who patronize the branch; each individual forms expectations about these waiting times. We are interested in the distribution of the individuals between the branches in an equilibrium in which each individual's expectations are correct.

13.1.2 Model

We model the street along which the individuals live as the interval $[0, 1]$; the bank branches are located at the points 0 and 1. The set of individuals is the

interval $[0, 1]$, with the interpretation that individual z resides at point z . Thus for each $z \in [0, 1]$, the fraction z of individuals reside to the left of z and the fraction $1 - z$ reside to the right of z . The assumption that the set of individuals is infinite aims to capture formally a situation in which the number of individuals is very large and each individual's behavior has a negligible effect on the waiting times in the branches, even though these waiting times are determined by the aggregate behavior in the population.

The waiting time in each branch depends on the number of individuals who use that branch. Specifically, if the fraction of individuals who use branch j (i.e. the branch located at j , which is 0 or 1) is n_j , then the waiting time in that branch is $f_j(n_j)$. We assume that each function f_j is increasing and continuous, with $f_j(0) = 0$ (i.e. if there are no customers in a branch, the waiting time in that branch is zero).

We assume, for simplicity, that an individual's travel time from x to branch z is the distance $d(z, x) = |z - x|$ between x and z . Every individual prefers the branch for which the sum of the travel time and the waiting time is smallest.

Definition 13.1: Service economy

A service economy $\langle B, I, (f_j)_{j \in B}, d \rangle$ consists of

branches

a set $B = \{0, 1\}$

individuals

a set $I = [0, 1]$

waiting time technology

continuous increasing functions $f_j : [0, 1] \rightarrow \mathbb{R}$ with $f_j(0) = 0$ for $j = 0, 1$, where $f_j(n_j)$ is the waiting time at branch j when the fraction of individuals who choose branch j is n_j

preferences

each individual $i \in I$ prefers a smaller loss to a larger one, where the loss from choosing branch j when t_j is the waiting time in that branch is $d(i, j) + t_j$, where $d(i, j) = |i - j|$.

Note that the bank branches are not decision-makers in this model: their locations and service technologies are fixed. The only decision-makers are the individuals.

13.1.3 Equilibrium

We define an equilibrium in the spirit of competitive equilibrium. Each individual has beliefs about the waiting times and assumes that her action does not

affect these waiting times. This assumption is analogous to our earlier assumption when defining competitive equilibrium that consumers and producers take prices as given, ignoring the effect of their own actions on the prices. Each individual chooses the branch that minimizes the time she spends dealing with the branch, given her beliefs about the waiting times. In equilibrium the individuals' beliefs are correct. Behind this definition is the assumption that agents' holding incorrect beliefs is a source of instability in the interaction; for stability, we need not only the individuals' actions to be optimal but also their beliefs to be correct.

A candidate for equilibrium consists of two numbers, t_0 and t_1 , the individuals' (common) beliefs about the waiting times in the branches, and a function $l : [0, 1] \rightarrow \{0, 1\}$, assigning to every individual at point x the branch $l(x)$ (either 0 or 1) that she chooses.

To be an equilibrium, a candidate has to satisfy two conditions.

- The decision of each individual is optimal given her beliefs about the waiting times in the branches.
- The individuals' decisions and beliefs are consistent in the sense that the belief about the waiting time in each branch is correct, given the service technology and the fraction of individuals who select that branch.

Definition 13.2: Equilibrium of service economy

An *equilibrium* of the **service economy** $\langle B, I, (f_j)_{j \in B}, d \rangle$ is a pair $((t_0, t_1), l)$, consisting of a pair of numbers (t_0, t_1) (the waiting times in the branches) and a function $l : I \rightarrow B$ (an assignment of each $x \in I$ to a branch), such that

optimality of individuals' choices

$$\begin{aligned} l(x) = 0 &\Rightarrow x + t_0 \leq (1 - x) + t_1 \\ l(x) = 1 &\Rightarrow (1 - x) + t_1 \leq x + t_0 \end{aligned}$$

(each individual is assigned to a branch for which the travel time plus waiting time for that branch is at most the travel time plus waiting time for the other branch)

consistency

$$t_j = f_j(\alpha(l, j)) \text{ for each } j \in B$$

where $\alpha(l, j)$ is the fraction of individuals assigned to branch j by the function l .

13.1.4 Analysis

We now prove the existence of an equilibrium in this model, characterize it, and show that it is Pareto stable. We start by showing that there is a unique point z^* such that if all individuals to the left of z^* use branch 0 and all individuals to the right of z^* use branch 1 then individual z^* is indifferent between the two branches.

Lemma 13.1

There is a unique number z^* such that $z^* + f_0(z^*) = 1 - z^* + f_1(1 - z^*)$.

Proof

The function $z + f_0(z)$ is continuous and increasing in z and takes the value 0 at the point 0 and the value $1 + f_0(1)$ at the point 1. The function $1 - z + f_1(1 - z)$ is continuous and decreasing in z and takes the value $1 + f_1(1)$ at 0 and the value 0 at 1. So the graphs of the functions have a unique intersection.

Next we show that for any expected waiting times, if for an individual at x branch 0 is at least as good as branch 1, then all individuals to the left of x prefer branch 0 to branch 1 (and analogously for an individual for whom branch 1 is at least as good as branch 0).

Lemma 13.2

For any pair of expected waiting times, if branch 0 is at least as good as branch 1 for an individual at x then branch 0 is better than branch 1 for every individual y with $y < x$, and if branch 1 is at least as good as branch 0 for an individual at x then branch 1 is better than branch 0 for every individual y with $y > x$.

Proof

Denote by t_0 and t_1 the expected waiting times in the branches. For branch 0 to be at least as good as branch 1 for an individual at x we need

$$t_0 + d(x, 0) \leq t_1 + d(x, 1).$$

If $y < x$ then $d(y, 0) < d(x, 0)$ and $d(y, 1) > d(x, 1)$, so that $t_0 + d(y, 0) < t_1 + d(y, 1)$. A similar argument applies to the other case.

We can now prove the existence and uniqueness of an equilibrium in a service economy.

Proposition 13.1: Equilibrium of service economy

Every **service economy** has a unique **equilibrium** (up to the specification of the choice at one point).

Proof

We first show that every service economy has an equilibrium. Let z^* be the number given in [Lemma 13.1](#). Let $(t_0^*, t_1^*) = (f_0(z^*), f_1(1 - z^*))$ and let l^* be the function that assigns 0 to all individuals in $[0, z^*]$ and 1 to all individuals in $(z^*, 1]$. We now argue that $((t_0^*, t_1^*), l^*)$ is an equilibrium.

Optimality of individuals' choices

Individual z^* is indifferent between the two branches since $z^* + t_0^* = z^* + f_0(z^*) = 1 - z^* + f_1(1 - z^*) = 1 - z^* + t_1^*$ (using the definition of z^*). By [Lemma 13.2](#), all individuals on the left of z^* prefer 0 to 1 and all on the right of z^* prefer branch 1 to 0.

Consistency

The proportion $\alpha(l^*, 0)$ of individuals who choose branch 0 is z^* . Therefore $t_0^* = f_0(z^*) = f_0(\alpha(l^*, 0))$. Similarly, the proportion $\alpha(l^*, 1)$ of individuals who choose branch 1 is $1 - z^*$, so that $t_1^* = f_1(1 - z^*) = f_1(\alpha(l^*, 1))$.

We now show that the equilibrium is unique. First note that a service economy has no equilibrium in which one branch is not used since if there were such an equilibrium, the waiting time at the unused branch would be 0 while the waiting time at the other branch would be positive, and hence individuals who are located close to the unused branch would prefer that branch to the other one.

Let $((t_0, t_1), l)$ be an equilibrium. By [Lemma 13.2](#), there is a point z such that all individuals to the left of z choose 0 and all individuals to the right of z choose 1. Thus an individual at z is indifferent between the branches, so that $z + t_0 = 1 - z + t_1$, and hence $z = z^*$ by [Lemma 13.1](#). Therefore l is identical to l^* up to the assignment at z^* . By the consistency condition for equilibrium, $t_0 = f_0(z^*)$ and $t_1 = f_1(1 - z^*)$.

We now define the notion of Pareto stability for a service economy and show that the equilibrium of such an economy is **Pareto stable**.

Definition 13.3: Pareto stability

Consider a **service economy** $\langle B, I, (f_j)_{j \in B}, d \rangle$. For any assignment l and individual $x \in I$ define $L_x(0, l) = x + f_0(\alpha(l, 0))$, the loss of x from choosing 0 given that all other individuals behave according to l . Similarly define $L_x(1, l) = 1 - x + f_1(\alpha(l, 1))$.

An assignment l is *Pareto stable* if there is no assignment l' that *Pareto dominates* l in the sense that $L_x(l'(x), l') \leq L_x(l(x), l)$ for all $x \in I$, with strict inequality for some $x \in I$.

Proposition 13.2: Pareto stability of equilibrium of service economy

Every equilibrium of a **service economy** is Pareto stable.

Proof

Let $((t_0^*, t_1^*), l^*)$ be an equilibrium of the **service economy** $\langle B, I, (f_j)_{j \in B}, d \rangle$. Let l' be an assignment. If the proportions of individuals at each branch are the same in l^* and l' , then the waiting times induced by the two assignments are the same. Since all individuals make the optimal choices in l^* , the assignment l' does not Pareto dominate l^* .

If more individuals are assigned to branch 0 (say) by l' than l^* , then some individuals who are assigned to branch 1 by l^* are assigned to branch 0 by l' . In the equilibrium such individuals like branch 1 at least as much as branch 0. Under l' , branch 0 is less attractive for each of them since the waiting time at that branch is greater than it is under l^* . Hence l' does not Pareto dominate l^* .

13.2 Asymmetric information and adverse selection**13.2.1 Introduction**

Second-hand cars of a particular model may differ substantially in quality. Each owner knows the quality of her car, but no buyer knows the quality of any given car. Because cars are indistinguishable to buyers, the price of every car is the same. Each owner decides whether to offer her car for sale, given this price. The decision of each potential buyer depends on her expectation of the quality of the cars offered for sale. A buyer may believe that the quality of the cars offered for sale is low, because owners of high-quality cars are not likely to want to sell, given the uniform price. The fact that the cars selected for sale by the owners have low quality is often called *adverse selection*.

13.2.2 Model

The set of individuals in the market consists of a finite set S of owners and a larger finite set B of potential buyers. Each $i \in S$ owns a car of quality $Q(i) \in (0, 1]$, which she knows. The utility of an owner of a car of quality q is q if she keeps it and p if she sells it at the price p . Each potential buyer obtains the utility $\alpha q - p$, where $\alpha > 1$, if she purchases a car of quality q at the price p , and the utility 0 if she does not purchase a car. The assumption that $\alpha > 1$ implies that mutually beneficial trade is possible: every car is valued more highly by every potential buyer than by its owner.

A potential buyer does not know and cannot determine the quality of any specific car before purchasing it, and no owner can credibly communicate the quality of her car to a potential buyer. Thus for a potential buyer, purchasing a car is a **lottery** with prizes equal to the possible qualities of the car. We assume that a buyer maximizes her expected utility, so her decision depends on her expectation \hat{q} of the quality of the cars for sale; she wishes to purchase a car if the amount she pays for it is less than $\alpha\hat{q}$.

Definition 13.4: Second-hand car market

A *second-hand car market* $\langle S, B, Q, \alpha \rangle$ consists of

owners

a finite set S , each member of which owns one car

buyers

a finite set B with $|B| > |S|$, each member of which buys at most one car

qualities

a function $Q : S \rightarrow (0, 1]$, where $Q(i)$ is the quality of the car owned by i

preferences

the owner of a car of quality q prefers to sell it if in exchange she gets an amount of money $p > q$ and prefers not to sell it if she gets an amount of money $p < q$

a potential buyer prefers to buy a car than not to do so if $\alpha\hat{q} > p$, prefers not to buy it if $\alpha\hat{q} < p$, and is indifferent between the two options if $\alpha\hat{q} = p$, where $\alpha > 1$ and p is the amount she pays and \hat{q} is her belief about the expected quality of the cars for sale.

13.2.3 Equilibrium

Two parameters determine the behavior of the buyers and owners: the price of a car and the belief of the potential buyers about the expected quality of the cars

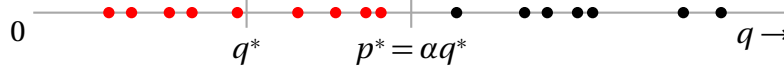


Figure 13.1 Equilibrium of a second-hand car market. Each small disk represents a car; the red ones are offered for sale.

for sale. An equilibrium consists of a price p^* , a (common) belief q^* of the potential buyers about the expected quality of cars for sale, a specification of the owners who offer their cars for sale, and a specification of the potential buyers who purchase cars, such that

- the decision of every owner and potential buyer is optimal, given p^* and q^*
- the number of cars offered for sale is equal to the number of buyers who wish to purchase a car
- if at least one car is traded, the buyers' belief about the expected quality of the cars offered for sale is correct (if there is no trade the belief is not restricted).

Definition 13.5: Equilibrium of second-hand car market

An *equilibrium* (p^*, q^*, S^*, B^*) of a **second-hand car market** (S, B, Q, α) consists of a number $p^* \geq 0$ (the price of a car), a number $q^* \geq 0$ (the potential buyers' common belief about the expected quality of the cars offered for sale), a set $S^* \subseteq S$ (the set of owners who offer their cars for sale), and a set $B^* \subseteq B$ (the set of potential buyers who purchase a car) such that

optimality of choices

for potential buyers: if $B^* \neq \emptyset$ then $p^* \leq \alpha q^*$ and if $B \setminus B^* \neq \emptyset$ then $p^* \geq \alpha q^*$

for owners: if $i \in S^*$ then $p^* \geq Q(i)$ and if $i \in S \setminus S^*$ then $p^* \leq Q(i)$

consistency

$|S^*| = |B^*|$ (the number of owners who sell their cars is equal to the number of potential buyers who buy a car)

if $S^* \neq \emptyset$ then $q^* = \sum_{i \in S^*} Q(i) / |S^*|$, the average quality of the cars owned by the members of S^* (the potential buyers' belief about the expected quality of the cars offered for sale is correct).

An equilibrium in which $\emptyset \subset B^* \subset B$, so that $p^* = \alpha q^*$, is illustrated in [Figure 13.1](#).

13.2.4 Analysis

We now show that every second-hand car market has an equilibrium in which trade occurs (the set of owners who sell their cars is nonempty).

Proposition 13.3: Equilibrium of second-hand car market

Let $\langle S, B, Q, \alpha \rangle$ be a **second-hand car market**. Name the owners so that $S = \{s_1, \dots, s_{|S|}\}$ with $Q(s_1) \leq Q(s_2) \leq \dots \leq Q(s_{|S|})$. The market has an **equilibrium** (p^*, q^*, S^*, B^*) with $S^* \neq \emptyset$. In any equilibrium the quality of every car that is sold is no greater than the quality of every other car.

Proof

For $m = 1, \dots, |S|$, let $A(m)$ be the average quality of the m lowest quality cars: $A(m) = \sum_{i=1}^m Q(s_i)/m$. Given $\alpha > 1$, we have $\alpha A(1) > Q(s_1)$. Let m^* be the maximal m for which $\alpha A(m) \geq Q(s_m)$. Let $p^* = \alpha A(m^*)$, $q^* = A(m^*)$, and $S^* = \{s_1, \dots, s_{m^*}\}$; let B^* be a subset of B with m^* members. Then (p^*, q^*, S^*, B^*) is an equilibrium. To verify the optimality of the individuals' choices, note that $p^* = \alpha q^* = \alpha A(m^*) \geq Q(s_{m^*}) \geq Q(s_m)$ for every $m \leq m^*$, so that each owner s_1, \dots, s_{m^*} optimally sells her car. Also $p^* = \alpha A(m^*) \leq \alpha A(m^* + 1) < Q(s_{m^*+1}) \leq Q(s_m)$ for all $m \geq m^* + 1$, so that each owner $s_{m^*+1}, \dots, s_{|S|}$ optimally does not sell her car. Each potential buyer is indifferent between buying and not buying a car since $\alpha q^* = p^*$.

The last claim in the proposition follows from the optimality of the owners' equilibrium choices. The quality of the cars of owners who sell is at most p^* and the quality of the other owners' cars is at least p^* .

Every second-hand car market has also an equilibrium in which no car is traded. Let p^* be a positive number less than $Q(s_1)$, the lowest quality, and let q^* be such that $\alpha q^* < p^*$. Then $(p^*, q^*, \emptyset, \emptyset)$ is an equilibrium: no potential buyer is willing to pay p^* for a car, given her belief that the average quality of the cars for sale is q^* , and no owner has a car whose quality is low enough to justify her selling it for p^* . In this equilibrium, the potential buyers expect that the average quality of cars for sale is less than the lowest quality of all owners' cars. Note that the definition of equilibrium does not restrict the belief of the potential buyers when no owner offers a car for sale. We might regard the belief q^* that we have assumed to be unreasonable. For example, if potential buyers know the range of qualities of the owners' cars, then their expectation should reasonably lie within this range, in which case an equilibrium in which no trade occurs does not exist.

Note that the equilibrium constructed in the proof of [Proposition 13.3](#) is not Pareto stable unless $S^* = S$. If $S^* \subset S$, suppose that the owner of a car of quality q who has not sold the car transfers it to a potential buyer who has not purchased a car, in exchange for an amount of money between q and αq . Then both the owner and the buyer are better off.

For some second-hand car markets, in all equilibria with trade only the lowest quality car is traded. Suppose for example that the set of car qualities is $\{1, 2, \dots, |S|\}$ and $\alpha < \frac{4}{3}$. In an equilibrium there is a number m^* such that S^* consists of the owners of cars with qualities $1, 2, \dots, m^*$ and $m^* \leq \alpha q^*$, where q^* is the average quality of the cars for sale, which is $\frac{1}{2}(1 + m^*)$. That is, $m^* \leq \frac{1}{2}\alpha(1 + m^*) < \frac{2}{3}(1 + m^*)$, which is satisfied only by $m^* = 1$.

13.3 A fishing economy

13.3.1 Introduction

A community of fishers and consumers lives near a lake. Each fisher decides how many fish to catch and each consumer decides how many fish to buy, given the price of fish. The cost of catching fish increases with the number of fish caught. In an equilibrium, the total amount of fish the fishers decide to catch is equal to the total amount the consumers decide to buy. Will the fishers catch too much in the sense that if they reduced their catch the price would adjust in such a way that everybody would be better off?

13.3.2 Model

The set of individuals in the economy consists of a set I of consumers and a set J of fishers. Each fisher decides how many fish to catch, up to a limit of L . If the total amount of fish caught by all fishers is T then the cost for a fisher to catch x fish is $c(T)x$, where c is a continuous, increasing function with $c(0) = 0$. That is, the larger is the total catch the more costly it is to fish. Each consumer decides how much fish to consume, up to a limit of one unit. Each consumer's preferences are represented by the function $v x + m$, with $v > 0$, where m is the amount of money she has and x is the amount of fish she consumes.

To make the main point of this section we analyze the model under the additional assumptions that (i) $c(|J|L) > v$ (if all fishers operate at full capacity then their unit cost exceeds the value of a unit to consumers), (ii) $c(0) < v$ (if all fishers are idle then their unit cost is less than the value of a unit to the consumers), and (iii) $|J|L \leq |I|$ (if all fishers operate at full capacity, their total output is less than the maximum possible total amount the consumers can consume).

Definition 13.6: Fishing economy

A fishing economy $\langle I, J, v, L, c \rangle$ consists of

consumers

a finite set I

fishers

a finite set J

consumers' preferences

a number $\nu > 0$, the consumers' monetary equivalent of a unit of fish, so that each consumer's preferences are represented by the utility function $\nu x + m$, where m is the amount of money the consumer has and $x \in [0, 1]$ is the amount of fish she consumes

fishers' technology

a number L with $0 < L \leq |I|/|J|$ and an increasing and continuous function $c : [0, |J|L] \rightarrow \mathbb{R}$ with $c(0) < \nu$ and $c(|J|L) > \nu$ (a fisher can catch up to L units of fish and one who catches y units incurs the cost $c(T)y$ when the total amount of fish caught by all fishers is T).

13.3.3 Equilibrium

A candidate for an equilibrium of a fishing economy consists of a price for a unit of fish, the fishers' common expectation about the unit cost of fishing, the amount of fish that each fisher decides to catch, and the amount of fish chosen by each consumer, such that

- every fisher chooses the amount of fish she catches to maximize her profit given the price and her expectation of the cost of fishing
- every consumer chooses her consumption optimally given the price
- the expectations of the fishers about the cost of fishing are correct
- the total amount of fish caught is equal to the total amount the consumers choose to consume.

Definition 13.7: Competitive equilibrium of fishing economy

A *competitive equilibrium* (p^*, c^*, y^*, x^*) of the **fishing economy** $\langle I, J, \nu, L, c \rangle$ consists of a positive number p^* (the price of a unit of fish), a non-negative number c^* (the fishers' belief about the unit cost of fishing), a non-negative number y^* (the amount of fish caught by each fisher), and a non-negative number x^* (the amount of fish chosen by each consumer) such that

optimality of choices

for consumers: x^* maximizes the utility $\nu x - p^*x$ over $[0, 1]$

for fishers: y^* maximizes the profit $p^*y - c^*y$ over $[0, L]$

feasibility

$|I|x^* = |J|y^*$ (the total amount of fish consumed is equal to the total amount of fish caught)

consistency

$c^* = c(|J|y^*)$ (the fishers' expectation about the unit fishing cost is correct).

13.3.4 Analysis

Proposition 13.4: Competitive equilibrium of fishing economy

A **fishing economy** $\langle I, J, v, L, c \rangle$ has a unique **competitive equilibrium** (p^*, c^*, y^*, x^*) , in which $p^* = v = c^* = c(|J|y^*)$ and $|I|x^* = |J|y^*$.

Proof

First, given $c(0) < v$, $c(|J|L) > v$, and the continuity of c there exists a number y^* such that $c(|J|y^*) = v$. Now, given that $c(|J|y^*) = v$, our assumptions that $c(|J|L) > v$ and c is increasing imply that $y^* < L$ and our assumption that $L \leq |I|/|J|$ implies that $x^* < 1$. The tuple (p^*, c^*, y^*, x^*) is a competitive equilibrium because all consumers and fishers are indifferent between all their possible actions, total production is equal to total consumption, and the fishers' expectation about the unit cost is correct.

To prove that the economy has no other equilibrium, suppose that (p', c', y', x') is an equilibrium.

If $p' > v$ then the optimal choice of every consumer is 0, so that $x' = y' = 0$. But then $c' = c(0) < v$, so that the optimal choice of every fisher is L , violating feasibility.

If $p' < v$ then the optimal choice of every consumer is 1, so that $x' = 1$ and by the feasibility condition $y' = |I|/|J|$. By the consistency condition $c' = c(|I|)$ and by our assumption that $|J|L \leq |I|$ we have $c(|I|) \geq c(|J|L) > v$, so that catching a positive amount of fish is not optimal for any fisher.

Therefore $p' = v$. It now suffices to show that $c' = p'$, since then by consistency we have $v = c(|J|y')$ and by feasibility $|J|y' = |I|x'$. If $c' > p'$ then the optimality of the fishers' choices implies that $y' = 0$; hence $x' = 0$, so that the optimality of the consumers' choices requires $p' \geq v$. But now by consistency $c' = c(0) < v$, a contradiction. A similar argument shows that $c' < p'$ is not possible.

A competitive equilibrium outcome is not **Pareto stable**, by the following argument. Let (p^*, c^*, y^*, x^*) be a competitive equilibrium. The utility of each consumer is $v x^* - p^* x^* = 0$ and the profit of each fisher is $p^* y^* - c(|J| y^*) y^* = 0$. Now consider \hat{y} and \hat{k} with $0 < \hat{y} < y^*$ and $c(|J|\hat{y}) < \hat{k} < v$. The production-consumption plan in which each fisher catches \hat{y} fish and receives $\hat{k}\hat{y}$ units of money and each consumer receives $\hat{y}|J|/|I|$ fish and pays $\hat{k}\hat{y}|J|/|I|$ generates positive utility to all consumers and positive profits to all fishers.

This model is used by many economists (including MJO, but not AR) to argue that a tax-redistribution scheme can make all agents (consumers and fishers) better off. Assume that each fisher has to pay a tax $t = v - c(|J|\hat{y})$ per unit of fish caught (where $0 < \hat{y} < y^*$), so that in equilibrium the unit cost for a fisher is $c^* + v - c(|J|\hat{y})$. This tax changes the unit cost of fishing when the total amount of fish caught is T from $c(T)$ to $d(T) = c(T) + v - c(|J|\hat{y})$, so that $d(|J|\hat{y}) = v$. Thus **Proposition 13.4** implies that the economy with the tax has a unique equilibrium, in which each fisher catches \hat{y} fish, the price paid by consumers is v , and each consumer purchases $\hat{y}|J|/|I|$ fish. In this equilibrium the utility of every consumer and the profit of every fisher is zero. The taxes collected can be distributed among the consumers and producers to make every consumer's utility and every fisher's profit positive.

Problems

1. Service economy.

- a. Compare the equilibrium of the **service economy** $\langle B, I, (f_j)_{j \in B}, d \rangle$ with the equilibrium of the service economy that differs only in that f_0 is replaced by \hat{f}_0 with $\hat{f}_0(x) < f_0(x)$ for all $x > 0$ (branch 0 becomes more efficient). Show that more individuals use branch 0 in an equilibrium of the modified economy than in an equilibrium of the original economy.
- b. Show that if branch 0 is more efficient than branch 1 in the sense that $f_0(x) < f_1(x)$ for every $x > 0$, then in equilibrium the waiting time in branch 1 is larger than it is in branch 0.
- c. Some evidence suggests that some people exaggerate their estimate of the time they spend in activities like going to a bank. (See for example **Jones and Hwang 2005**.) Assume that an individual who spends the total amount of time t acts as if this total time is λt , with $\lambda > 1$. How does the equilibrium change?
- d. How does the equilibrium change if individuals exaggerate only the waiting time in a branch, not the transportation time?

2. *Total loss in equilibrium of service economy.* Consider a [service economy](#). Suppose that all individuals to the left of z use branch 0 and the remainder use branch 1. Then the total time spent by the individuals is

$$\int_0^z [x + f_0(z)] dx + \int_z^1 [(1-x) + f_1(z)] dx.$$

Explain why the equilibrium may not (and typically does not) minimize the total time spent by all individuals even though we know from [Proposition 13.2](#) that the equilibrium is Pareto stable. (If you wish, just calculate the equilibrium for the service economy with $f_0(x) = x$ and $f_1(x) = 2x$ and show that the assignment of individuals to branches that minimize the total loss differs from the equilibrium allocation.)

3. *Fund-raising party.* Each of the 1,200 participants at a fund-raising event can choose a raffle ticket marked L or H . One ticket marked L is randomly chosen and its holder is given the prize L , and one ticket marked H is randomly chosen and its holder is given the prize H , where $0 < L < H$. The preferences of each individual i over the set of lotteries are represented by the expected value of a Bernoulli utility function u^i with $u^i(0) = 0$, $u^i(H) = 1$, and $u^i(L) = \nu$, where $0 < \nu < 1$.

a. Formulate an equilibrium concept in the spirit of this chapter.

b. What is the equilibrium if $\nu = \frac{1}{3}$?

4. *Matching.* Individuals are divided into a members of type A and b members of type B , where $a \geq b$. Each individual wishes to be matched with an individual of the other type. An individual can be matched with only one other individual. Matches can occur in two possible venues, 1 and 2. Each individual chooses one of these venues. Given that α individuals of type A and β individuals of type B choose a venue, the probability of a type A individual being matched at that venue is $\min\{\alpha, \beta\}/\alpha$ and the probability of a type B individual being matched is $\min\{\alpha, \beta\}/\beta$. Each individual chooses a venue to maximize the probability she is matched.

A profile is a list (a_1, b_1, a_2, b_2) of nonnegative real numbers for which $a_1 + a_2 = a$ and $b_1 + b_2 = b$, with the interpretation that a_i and b_i are the numbers of type A and type B individuals who choose venue i . For simplicity, we do not require these numbers to be integers. A candidate for equilibrium is a profile (a_1, b_1, a_2, b_2) and a vector of nonnegative numbers (p_1, p_2, q_1, q_2) with $p_1 + p_2 = 1$ and $q_1 + q_2 = 1$, where p_i is the probability a type A individual

assigns to being matched in venue i and q_i is the probability that a type B individual assigns to being matched in venue i . A candidate is an equilibrium if the following two conditions are satisfied.

Optimality

If some type A individual is assigned to venue i ($a_i > 0$) then $p_i \geq p_j$, where $j \neq i$, and if some type B individual is assigned to venue i ($b_i > 0$) then $q_i \geq q_j$.

Consistency

We have $p_i = \min\{1, b_i/a_i\}$ and $q_i = \min\{1, a_i/b_i\}$ for $i = 1, 2$. (Define $\min\{1, 0/0\} = 0$ and for every $x \neq 0$ define $\min\{1, x/0\} = 1$.)

- a. Show that any profile (a_1, b_1, a_2, b_2) satisfying $b_1/a_1 = b_2/a_2 = b/a$ together with the vector (p_1, p_2, q_1, q_2) with $p_i = b/a$ and $q_i = 1$ for $i = 1, 2$ is an equilibrium.
 - b. Characterize all the equilibria for which individuals of each type choose each venue.
 - c. Find an equilibrium in which every individual chooses venue 1.
5. *Health services.* Consider a market for health services in which there is a large number n of individuals, each with a large amount of money m . Each individual can purchase a quantity of health services. If individual i buys $y(i)$ units of health services, the probability that she survives is $\alpha(y(i), y^*)$, where y^* is the average level of health services obtained by all individuals. Individuals take y^* as given although it is influenced by their behavior. The function α is increasing and concave. Each individual aims to maximize the product of the amount of money she is left with and the probability of survival. That is, she chooses $y(i)$ to maximize $(m - p^*y(i))\alpha(y(i), y^*)$.
- a. Define an equilibrium. Write the equations that characterize an equilibrium in which all agents purchase a positive quality of health services. Assume that the function α is differentiable.
 - b. Explain why an equilibrium is not Pareto stable.

Notes

The model in [Section 13.2](#) is due to [Akerlof \(1970\)](#).

