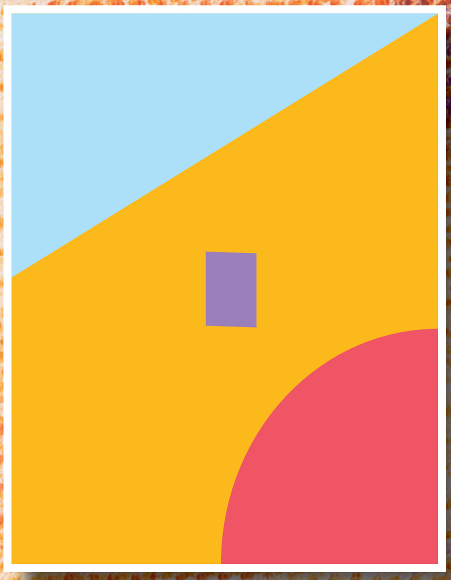
Models in Microeconomic Theory

Expanded Second Edition

Martin J. Osborne
Ariel Rubinstein



https://www.openbookpublishers.com

Updated digital material and resources associated with this book are available at https://www.openbookpublishers.com/product/1171#resources.

ISBN Paperback: 978-1-80511-121-4
ISBN Hardback: 978-1-80511-122-1
ISBN Digital (PDF): 978-1-80511-123-8
DOI: 10.11647/OBP.0361

This book is available in two versions, one that uses feminine pronouns and one that uses masculine pronouns. This version uses feminine pronouns.

Version: 2023.5.30 (s)

14 A market with asymmetric information

In this chapter we study an equilibrium concept that differs from the notions of competitive equilibrium discussed in Chapters 9–12. The models in the earlier chapters specify the precise set of economic agents who operate in the market, and an equilibrium specifies the terms of trade (prices) for which the aggregate demand and supply of these agents are equal. The model we study in this chapter does not explicitly specify the set of agents. As a consequence, the equilibrium notion is more abstract. A set of contracts is an equilibrium if no agent who offers a contract wants to withdraw it, and no agent can profit by adding a contract.

We illustrate the concept by applying it to a model central to the economics of information. The problems at the end of the chapter demonstrate the use of the concept to study other economic interactions.

14.1 Introductory model

To explain the logic of the solution concept, we start with a model of a simple labor market without asymmetric information. The market contains employers and workers. Employers post wage offers. Workers are identical, with productivity, if employed, equal to $v > 0$. Each worker either selects a posted offer that is best for her or, if no posted offer is better for her than being unemployed, does not select any offer. A worker who selects an offer is matched with the employer posting the offer; she produces v and receives the posted wage. The profit of an employer who pays a wage w to a worker is $v - w$.

We say that a wage offer (a nonnegative number) is optimal for a worker given the set W of offers if it is the highest wage in W. (We assume that not accepting any offer is equivalent to receiving a wage of 0.) An equilibrium is a finite set W of wage offers for which

I. every offer in W is optimal for a worker

II. no offer in W generates a loss to an employer who posts it

III. no offer $w \notin W$ is optimal for a worker, given the offers in $W \cup \{w\}$, and would yield an employer who posts it a positive profit.

Chapter of *Models in Microeconomic Theory* by Martin J. Osborne and Ariel Rubinstein. Version 2023.5.30 (s).
 https://doi.org/10.11647/OBP.0361.14

Condition I captures the idea that offers that are not accepted by any worker do not survive. Condition II requires that no offer that is accepted yields a loss to the employer who posts it. Condition III requires that no employer can post a new offer that is optimal for a worker and yields the employer a positive profit.

The notion of equilibrium differs from the ones we analyze in earlier chapters in that it does not specify the choices made by specific participants. An equilibrium is a set of acceptable contracts in the market. The equilibrium is silent about who makes which offer.

We claim that the set $\{v\}$, consisting of the single wage offer v, is an equilibrium. Workers optimally choose it, as it is better than not accepting an offer; it yields zero profit to an employer; and any new offer is either not accepted (if it is less than v) or is accepted (if it is greater than v) but yields negative profit to the employer who posts it.

In fact, $\{v\}$ is the only equilibrium. Let W^* be an equilibrium. If $W^* = \varnothing$ then any offer w with $0 < w < v$ is optimal given $W^* \cup \{w\} = \{w\}$ and yields an employer who posts it a positive profit, violating III. By I, if $W^* \neq \varnothing$ then W^* consists of a single offer, say w^*. By II, $w^* \leq v$. If $w^* < v$, then an offer w with $w^* < w < v$ is optimal for a worker given $W^* \cup \{w\}$ and yields a positive profit, violating III.

14.2 Labor market with education

Imagine a labor market in which employers do not know, before hiring workers, how productive they will be, but do know their educational backgrounds. If education enhances productivity, we might expect employers to be willing to pay higher wages to more educated workers. We study a model in which education does *not* affect productivity, but productivity is negatively related to the cost of acquiring education: the more productive a worker, the less costly it is for her to acquire education. Under this assumption, we might expect employers to be willing to pay a high wage to a worker with a high level of education because they believe that acquiring such an education is worthwhile only for high productivity workers. The model we study investigates whether such a relation between wages and education exists in equilibrium.

In the model there are two types of worker, H and L. When employed, a type H worker creates output worth v_H and a type L worker creates output worth v_L, where $0 < v_L < v_H$. The proportion of workers of type L in the population is α_L and the proportion of workers of type H is α_H, with $\alpha_H + \alpha_L = 1$. No employer knows the type of any given worker before hiring her.

If an employer offers a wage w and a worker who accepts the offer is of type H with probability γ_H and type L with probability γ_L then the employer obtains an expected profit of $\gamma_H v_H + \gamma_L v_L - w$. If a contract is simply a wage offer, then by the argument in the previous section the only equilibrium is $\{\overline{v}\}$, where $\overline{v} = \alpha_H v_H + \alpha_L v_L$, the expected productivity of a worker.

We now add education to the model. Each worker chooses a level of education, which does *not* affect her productivity. The cost of obtaining education is linear in the level of education and is higher for type L workers (who have lower productivity) than it is for type H workers. Specifically, the income of a type X worker with t years of education who is paid the wage w is $w - \beta_X t$, with $0 < \beta_H < \beta_L$.

An employer can observe a worker's education but not her productivity. A contract now specifies a wage and a minimal acceptable number of years of education.

Facing a set of contracts, an individual who is planning her career chooses her education level bearing in mind the maximal wage that this level allows her to obtain. Thus her decision is to choose one of the available contracts (or to choose not to be employed). We assume that each worker's preferences over contracts are lexicographic: her first priority is high income (taking into account the cost of the required level of education), and among contracts that yield her the same income, she prefers one with a lower educational requirement. Thus no worker is indifferent between any two contracts.

Definition 14.1: Labor market with asymmetric information

A *labor market with asymmetric information* is a list of numbers $(v_L, v_H, \alpha_L, \alpha_H, \beta_L, \beta_H)$, where $0 < v_L < v_H$, $\alpha_L \geq 0$, $\alpha_H \geq 0$, $\alpha_L + \alpha_H = 1$, and $0 < \beta_H < \beta_L$. The market consists of *employers* and *workers*. The workers are of two types.

Type L (fraction α_L)
Productivity v_L and cost β_L for each unit of education

Type H (fraction α_H)
Productivity v_H and cost β_H for each unit of education.

A *contract* is a pair (t, w) of nonnegative numbers; t is the number of units of education required for the job and w is the wage.

We now specify a worker's preferences over contracts.

Definition 14.2: Worker's preferences

In a labor market with asymmetric information $(v_L, v_H, \alpha_L, \alpha_H, \beta_L, \beta_H)$, the *income* of a worker of type X $(= H, L)$ who accepts the contract (t, w) is

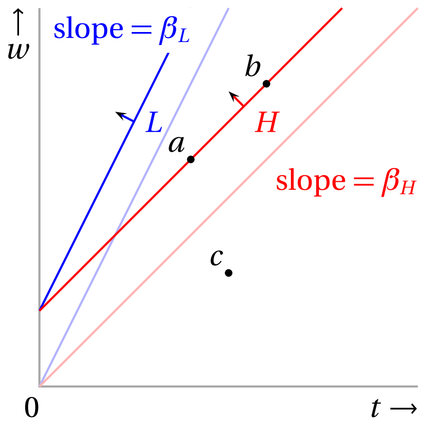


Figure 14.1 Iso-income lines for workers. Along each blue line the income of a worker of type L is constant, and along each red line the income of a worker of type H is constant. Income is higher along the darker lines. A type H worker prefers a to b because a requires less education and both contracts yield her the same income. If only c is offered, each type of worker prefers not to accept any offer, because c yields negative income.

$w - \beta_X t$. The preference relation $\succsim^X$ of a worker of type X over the set of contracts is lexicographic, giving first priority to larger income $w - \beta_X t$ and second priority to smaller values of the education requirement t.

For any set C of contracts the alternative that is *optimal given C for a worker of type X* ($= H, L$) is

$$\begin{cases} (t,w) \in C & \text{if } w - \beta_X t \geq 0 \text{ and } (t,w) \succsim^X (t',w') \text{ for all } (t',w') \in C \\ \phi & \text{if } w' - \beta_X t' < 0 \text{ for every } (t',w') \in C, \end{cases}$$

where ϕ means that the worker does not accept any contract.

Figure 14.1 shows iso-income lines for each type of worker. The blue lines belong to a type L worker; their slope is β_L. Each additional unit of education has to be compensated by an increase β_L in the wage to keep the income of such a worker the same. The red iso-income lines belong to a type H worker; their slope is β_H. Incomes for each type increase in a northwesterly direction: every worker prefers contracts with lower educational requirements and higher wages.

Given that the set of contracts offered is C, an employer who offers a contract $c = (t,w)$ expects a payoff that depends on the types of workers for whom c is optimal given C. If c is not optimal given C for any worker, the employer's payoff is zero; if c is optimal given C only for type X workers, her payoff is $v_X - w$, the profit from hiring a type X worker; and if c is optimal for all workers, her payoff is $\alpha_L v_L + \alpha_H v_H - w$.

Definition 14.3: Employer's payoff

In a labor market with asymmetric information $(v_L, v_H, \alpha_L, \alpha_H, \beta_L, \beta_H)$, for any set C of contracts and any $c = (t, w) \in C$, the *payoff* $\pi(c, C)$ of an employer who offers the contract c when C is the set of posted contracts is

$$\begin{cases} 0 & \text{if } c \text{ is not optimal given } C \text{ for either type of worker} \\ v_X - w & \text{if } c \text{ is optimal given } C \text{ only for type } X \text{ workers} \\ \alpha_L v_L + \alpha_H v_H - w & \text{if } c \text{ is optimal given } C \text{ for both types of worker.} \end{cases}$$

The payoff of an employer who does not offer a contract is 0.

Equilibrium An equilibrium is a finite set C^* of contracts for which (I) every contract in C^* is optimal for at least one type of worker given C^*, (II) no contract in C^* yields a negative payoff to an employer, and (III) no contract $c \notin C^*$ that is optimal for at least one type of worker given $C^* \cup \{c\}$ yields a positive payoff for an employer.

Note that this notion of equilibrium reflects an assumption that an employer who considers offering a new contract correctly anticipates the types of workers for whom the contract is optimal given the other contracts offered.

Definition 14.4: Equilibrium of labor market

An *equilibrium* of a labor market with asymmetric information $(v_L, v_H, \alpha_L, \alpha_H, \beta_L, \beta_H)$ is a finite set C^* of contracts such that

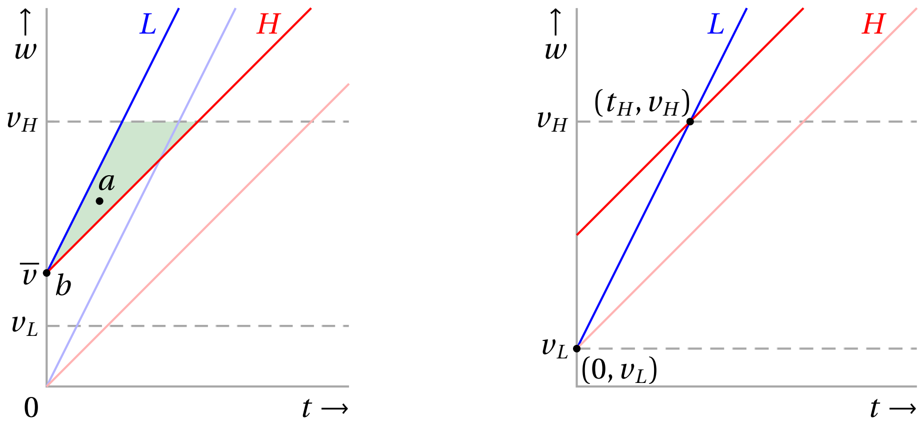
I. each $c \in C^*$ is optimal given C^* for at least one type of worker

II. if $c \in C^*$, then $\pi(c, C^*) \geq 0$ (no employer wants to withdraw a contract)

III. if $c \notin C^*$ and c is optimal given $C^* \cup \{c\}$ for some type of workers, then $\pi(c, C^* \cup \{c\}) \leq 0$ (no employer wants to add a contract).

An equilibrium C^* for which the same alternative is optimal given C^* for both types of worker is a *pooling equilibrium*. An equilibrium for which a different alternative is optimal given C^* for each type is a *separating equilibrium*.

We first argue that the set consisting solely of the contract $b = (0, \overline{v}) = (0, \alpha_H v_H + \alpha_L v_L)$ is not an equilibrium. The reason is that the contract a in Figure 14.2a, like any contract in the area shaded green, is optimal given the set



(a) Illustration of the argument that $\{(0,\overline{v})\}$ is not an equilibrium.

(b) The contracts offered in an equilibrium (Proposition 14.1), if one exists.

Figure 14.2

$\{a, b\}$ for a type H worker but not for a type L worker. Thus, given that the wage in a is less than v_H, an employer who offers a when the only other contract is b obtains a positive payoff, violating condition III in Definition 14.4.

The next result shows more generally that a labor market with asymmetric information has no pooling equilibrium and that an equilibrium, if one exists, is separating, containing two contracts, one of which is optimal for each type of worker. These contracts are illustrated in Figure 14.2b. The contract optimal for a type L worker entails a wage equal to her productivity, v_L, and no education ($t = 0$). The contract optimal for a type H worker also pays a wage equal to her productivity, v_H, but requires enough education that a type L worker is not better off choosing it.

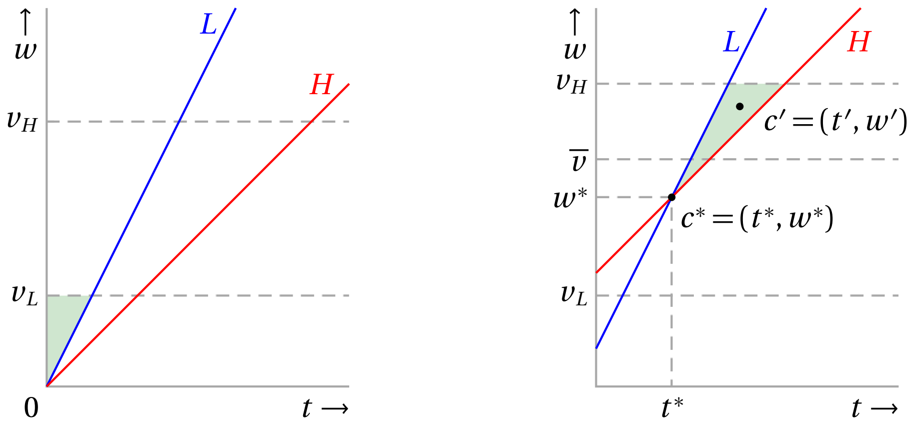
Proposition 14.1: Characterization of equilibrium of labor market

If a labor market with asymmetric information $(v_L, v_H, \alpha_L, \alpha_H, \beta_L, \beta_H)$ has an equilibrium C^*, then $C^* = \{c_L^*, c_H^*\}$ where $c_L^* = (0, v_L)$, $c_H^* = (t_H, v_H)$, and t_H satisfies $v_H - \beta_L t_H = v_L$. Given C^*, the contract c_L^* is optimal for a type L worker and c_H^* is optimal for a type H worker.

Proof

Consider an equilibrium C^*.

Step 1 *The action ϕ (not accepting any offer) is not optimal for either type of worker given C^* and thus in particular C^* is not empty.*



(a) Step 1 of the proof of Proposition 14.1. (b) Step 2 of the proof of Proposition 14.1.

Figure 14.3

Proof. Suppose that ϕ is optimal for some type X worker given C^*. Then no contract in the region shaded green in Figure 14.3a is in C^* (because if it were it would be better than ϕ for type X given C^*). Thus any such contract c is optimal for type X given $C^* \cup \{c\}$. Whether X is L or H, the contract c yields a positive payoff for an employer (the wage is less than v_L), and thus violates condition III. $\lhd$

Step 2 *C^* is not a pooling equilibrium.*

Proof. Assume C^* is a pooling equilibrium. By Step 1, C^* is nonempty, so some contract, say $c^* = (t^*, w^*)$, is optimal given C^* for both types, and by condition I, C^* contains no other contract.

Suppose that $w^* > \overline{v} = \alpha_H v_H + \alpha_L v_L$. Then $\pi(c^*, C^*) = \overline{v} - w^* < 0$, violating condition II.

Now suppose that $w^* \leq \overline{v}$. Consider a contract $c' = (t', w')$ in the green triangle in Figure 14.3b. That is, $w^* + \beta_H(t' - t^*) < w' < \min\{w^* + \beta_L(t' - t^*), v_H\}$. The contract c' is optimal given $\{c^*, c'\}$ for type H, and is not optimal given $\{c^*, c'\}$ for type L (who prefer c^*). Thus $\pi(c', \{c^*, c'\}) = v_H - w' > 0$, violating condition III. $\lhd$

Given Step 1, Step 2, condition I, and the fact that no worker is indifferent between any two contracts, C^* contains exactly two contracts, say $C^* = \{c_L, c_H\}$, where $c_L = (t_L, w_L)$ is optimal given C^* for type L workers and $c_H = (t_H, w_H)$ is optimal given C^* for type H workers. We now characterize these two contracts.

Step 3 *$w_X \le v_X$ for $X = H, L$.*

Proof. For an employer who offers the contract c_X, $\pi(c_X, C^*) = v_X - w_X$, so that $w_X \le v_X$ condition II. ◁

Step 4 $c_L = c_L^* = (0, v_L)$.

Proof. By Step 3, $w_L \le v_L$. If $w_L < v_L$ then the contract $c = (t_L, \frac{1}{2}(v_L + w_L))$ is optimal given $C^* \cup \{c\}$ for (at least) type L workers, so that $\pi(c, C^* \cup \{c\}) \ge v_L - \frac{1}{2}(v_L + w_L) = \frac{1}{2}(v_L - w_L) > 0$, violating condition III. Thus $w_L = v_L$.

If $t_L > 0$, let $c' = (t'_L, w'_L)$ with $t'_L < t_L$, $w'_L < w_L$, and $w'_L - \beta_L t'_L > w_L - \beta_L t_L$. That is, c' reduces the education requirement and the wage in such a way that the income of a type L worker increases. Then c' is optimal given $C^* \cup \{c'\}$ for at least type L workers, so that $\pi(c', C^* \cup \{c'\}) \ge v_L - w'_L > 0$, violating condition III. Thus $t_L = 0$. ◁

Step 5 *$w_H = v_H$ and c_L and c_H yield the same income for a type L worker, so that $v_H - \beta_L t_H = v_L$.*

Proof. By Step 3, $w_H \le v_H$. Given that c_H is optimal given C^* only for a type H worker and c_L is optimal only for a type L worker, c_H lies in the green region in Figure 14.4. If c_H is not c_H^* (the point at the intersection of the horizontal line $w = v_H$ and the line $w - \beta_L t = v_L$) then any contract c'_H in the interior of the dark green region is better for type H workers than c_H but worse for type L workers than c_L. Thus c'_H is optimal given $\{c_L, c_H, c'_H\}$ only for type H workers, so that $\pi(c'_H, \{c_L, c_H, c'_H\}) > 0$, violating condition III. Hence $w_H = v_H$ and $v_H - \beta_L t_H = v_L$, so that $c_H = c_H^*$. ◁

Whether the set of contracts specified in this result is in fact an equilibrium depends on the proportions of the types of workers in the population. Let $m = v_H - \beta_H t_H$ so that the contract $(0, m)$ yields type H workers the same income as does c_H^*.

If the proportion of type L workers is high enough that the average productivity in the entire population, $\overline{v}$, is less than m (as in Figure 14.5a), then C^* is an equilibrium, by the following argument.

- Each c_X^* is optimal for workers of type X.
- Each contract c_X^* yields a payoff of zero to an employer.
- Any contract $c = (t, w)$ that is optimal given $\{c_L^*, c_H^*, c\}$ only for type H workers is in the area shaded green in Figure 14.5a, so that $w > v_H$ and thus the contract does not yield a positive profit.

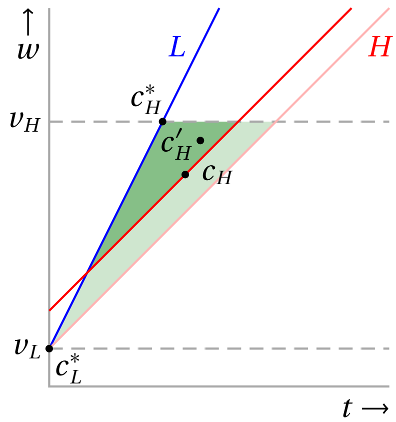


Figure 14.4 Step 5 of the proof of Proposition 14.1.

- Any contract $c = (t, w)$ that is optimal given $\{c_L^*, c_H^*, c\}$ for type L workers has $w > v_L$, so that if c is optimal only for type L workers it yields a negative profit.
- Any contract $c = (t, w)$ that is optimal given $\{c_L^*, c_H^*, c\}$ for both types of worker lies above the iso-income curve of a type H worker through c_H^* (the dark red line in Figure 14.5a), so that $w > m$; since $m > \overline{v}$ we have $w > \overline{v}$, so that c is not profitable.

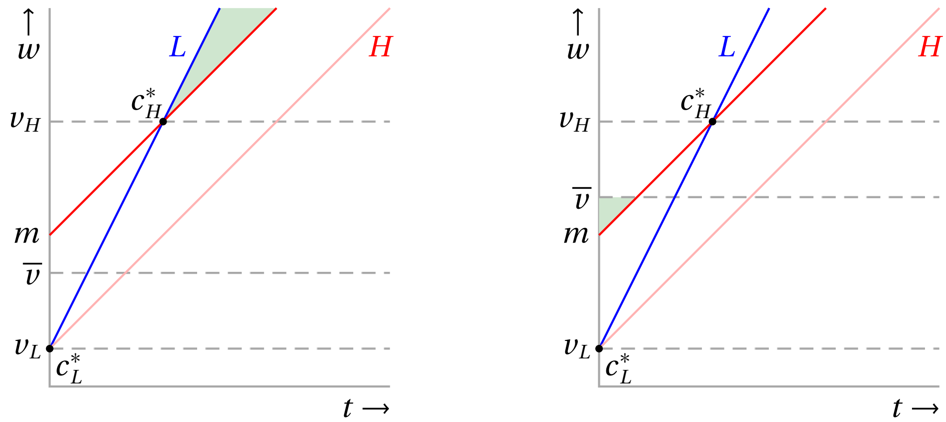
If, on the other hand, the proportion of workers of type H is large enough that the average productivity in the population exceeds m, then an employer who adds the contract $(0, m)$ (or any other contract in the green triangle in Figure 14.5b) attracts workers of both types and obtains a positive profit. Thus in this case the set C^* of contracts is not an equilibrium.

Comment

The model is related to the "handicap principle" in biology. This principle provides an explanation for phenomena like the long horns of male deer. The male deer signals his unobserved fitness (biological value) by wasting resources on useless horns. The usefulness of the signal depends on the fact that spending resources on useless horns is less costly for fitter animals. In the economic story, a worker signals her unobserved quality by obtaining education, which has no effect on her productivity but is less costly for workers with high productivity.

Problems

1. *Quality certificate.* A market contains producers, each of whom can produce one unit of a good. The quality of the good produced by half of the producers



(a) In this market, $\{c_H^*, c_L^*\}$ is a separating equilibrium.

(b) In this market, $\{c_H^*, c_L^*\}$ is not a separating equilibrium.

Figure 14.5 Existence or nonexistence of equilibria in labor markets with asymmetric information.

(type L) is low, and the quality of the good produced by the remaining half (type H) is high. Each producer knows the quality of her output and has no production cost.

The market contains also traders, each of whom can buy a unit of the good from a producer. If a trader buys a unit, she can sell it for the price 20 if it is high quality and for the price 10 if it is low quality. No trader can verify the quality of a good prior to purchasing it.

A producer can obtain a certificate that says that her output has high quality. The cost of such a certificate is 4 for type H and 12 for type L. (A type L producer has to bribe the agency who gives the certificate.)

Traders make offers. An offer has either the form $(+, p)$, a promise to pay p for a good with a certificate, or the form $(-, p)$, a promise to pay p for a good without a certificate. Traders maximize profits.

Each producer has to decide whether to accept one of the offers or to reject all offers (in which case her profit is 0). Producers maximize profits. A producer who is indifferent between two offers chooses the one without the certificate.

A candidate for equilibrium is a set of offers. Define a notion of equilibrium in the spirit of this chapter and characterize all equilibria.

2. *Sorting students.* Consider a world in which entrepreneurs offer education services to the students in a city. All students must choose a school (if one exists). Every student appreciates the closeness of a school to the city. There

are two styles of schools, A and B. A school is a pair (x, d), where x is the style and d is the distance from the city. (The notion of school is analogous to that of a contract in the body of the chapter.) For any value of d, every student prefers (A, d) to (B, d).

The students are of two types.

- A student of type 1 is willing to travel an extra 10 kilometers to get to an A-school. That is, $(A, d) \succsim^1 (B, d')$ if and only if $d \leq d' + 10$. A student of this type fits better at an A-school.
- A student of type 2 is willing to travel only an extra 5 kilometers to study in an A-school. That is, $(A, d) \succsim^2 (B, d')$ if and only if $d \leq d' + 5$. A student of this type fits better at a B-school.

If $(A, d) \sim^i (B, d')$, so that $d > d'$, then a student of type i chooses school B.

Assume that a new school is established only if it is expected that all the students who find it optimal fit its style. An existing school closes if no student attends it or if all students who find it optimal fit the other style of school. Note the following asymmetry: an existing school remains open if it attracts a mixed population whereas to be established, a new school has to expect to attract only students that fit its style.

Define a notion of equilibrium and characterize it.

Notes

The economic example in this chapter is based on Spence (1973) but the analysis follows Rothschild and Stiglitz (1976). The handicap principle is due to Zahavi (1975).