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BREAKING IMAGES

ICONOCLASTIC ANALYSES OF MATHEMATICS AND ITS EDUCATION

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5. Dehumanisation through mathematics

David Kollosche

Mathematics can be seen as a project of dehumanisation in the sense that it allows us to work with a disregard for personal uniqueness. While the word 'dehumanisation' has a negative connotation and invites us to study this property of mathematics carefully, dehumanisation is also one of the great strengths of mathematics and has proved invaluable for modern society. We will trace this field of tension along two indisputable ingredients of mathematics activity: calculation and logic. As there is enough literature praising mathematics, we allow ourselves to take a more critical stance towards dehumanisation through mathematics. We end with a sceptical discussion on whether mathematics can be rehumanised.

Dehumanisation

I will argue that mathematics is a project of *dehumanisation*.[1] What do I mean with that term? Dehumanisation has become a present concept for the analysis of the psychological and sociological phenomenon of denying people their full humanness, usually in order to justify practices of injustice, violence, and silencing. This understanding of the term dates back to Herbert C. Kelman's (1973) analysis of the mental configuration which allowed people to be well-educated and family-loving while, at the same time, committing some of the most devastating crimes in human history, especially in the Holocaust. Nick

1 For a discussion of dehumanisation through mathematics *education*, see Bishop (1988, especially pp. 12–13).

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Haslam (2015) provided a good overview of the research field that has developed on this basis, as does the freshly published *Routledge Handbook of Dehumanization* (Kronfeldner, 2021).

An older use of the term dates back to the work of Max Weber, one of the pioneers of sociology. In his monumental work *Economy and Society*, Weber (1978) argued that the bureaucratic organisation of administration is technically superior to any other form of administration because it is 'dehumanised' (*entmenschlicht*). Allow me to present the broader context of Weber's thoughts:

> Bureaucratization offers above all the optimum possibility for carrying through the principle of specializing administrative functions according to purely objective considerations. Individual performances are allocated to functionaries who have specialized training and who by constant practice increase their expertise. 'Objective' discharge of business primarily means a discharge of business according to *calculabe rules* and 'without regard for persons.' [...] When fully developed, bureaucracy also stands, in a specific sense, under the principle of *sine ira ac studio* [without anger and passion]. Bureaucracy develops the more perfectly, the more it is 'dehumanized,' the more completely it succeeds in eliminating from official business love, hatred, and all purely personal, irrational, and emotional elements which escape calculation. (p. 975)

Note that, first of all, Weber was not writing about people who become dehumanised. Instead, it is a form of administration, comprising a certain body of knowledge, specific practices and particular perspectives on social affairs, which are 'dehumanised'. What is this supposed to mean? Clearly, bureaucracy has never been a human being, which could suddenly come to be denied this status. However, administrative action is executed by people and affects people. The idea of bureaucracy is that the practice of administration is organised in ways which ensure that the individuality of both administrators and administered is denied. All the administrator is supposed to do is 'calculation', a mechanical processing of official affairs, while the administered is relevant only in terms of the data retrieved for the processing of a specific administrative act. Dehumanisation, here, is understood as a social practice which contributes to the denial of somebody's humanity.[2]

2 There is an interesting debate concerning the differences between the concepts of 'dehumanisation' and 'objectification' (Mikkola, 2021), but this is not the place to continue this debate.

Turning the focus to the people performing bureaucratic work opens up another perspective on dehumanisation. Weber (1978) noted that 'the spirit in which the ideal official conducts his office' is dominated by 'a spirit of formalistic impersonality'. Administrators have to work 'without hatred or passion, and hence without affection or enthusiasm' for their work. 'The dominant norms are concepts of straightforward duty without regard to personal considerations' and everybody 'is subject to formal equality of treatment' (p. 225). In consequence, the administrator has to work like a machine, has to behave in a way that could be called 'dehumanised'. That means that the *dehumanised practice* of bureaucratic administration does not only lead to the *dehumanisation of others*, first of all it requires a *dehumanisation of the self*. It appears that these dimensions of dehumanisation necessarily go together.

Note that Weber (1978) wrote that bureaucracy would be *technically* superior to other forms of administration (p. 973)! Apparently, he was careful enough to reserve some scepticism about the good of a fully dehumanised practice. This scepticism was well placed. To address an extreme example, mind Bauman's (1989) analysis that the highly demanding administrative organisation of the Holocaust was possible only because administrators worked in a demoralised, mechanical way. Despite his caution, Weber was a great admirer of bureaucracy which becomes clear when considering his historical situation. At his time, Weber witnessed a transformation from a poorly organised society, which suffered from poverty, starvation and extreme inequalities and in which support and rights depended largely on birth right, to a highly organised society, in which support and rights were allegedly equally distributed. Bureaucracy, then, was perceived as a tremendous step towards efficiency and equality. Obviously, it would be naïve to simply consider dehumanisation something good or bad. However, both the positive and the negative potential call for a closer analysis.

Mathematics

Alan Bishop (1988) brought up the concept of dehumanisation when discussing rationalism and objectism (the study of objects instead of actions) as cultural values of mathematics:

> So, once again we see, with objectism as with rationalism, an ideology which is in some sense dehumanised. Rationalism is about certain criteria of theories, divorced from their human creators, while objectism is based on inanimate objects and not on animate phenomena, such as humans. Mathematics favours an objective, rather than a subjective, view of reality. (p. 66)

Bishop's remark suggests that dehumanisation is a central ingredient of mathematics, and that dehumanisation through mathematics is a concern for understanding our culture. I want to depart from Bishop's remark and attempt to provide a more elaborated discussion of dehumanisation through mathematics.

My initial statement that mathematics is a project of dehumanisation could now be rephrased to claim that mathematics is a social practice which seeks to work with a disregard for personal uniqueness, with an emphasis on mechanical predictiveness. In this reading, my initial statement appears to be a negatively connotated account of something that is all too obvious: of course, there is an ideal that practices such as proving or calculating are independent of the individual. The belief that such practices are possible explains, to a large extent, the fascination for and use of mathematics. The technical possibility of this independence has been documented in the execution of such practices by electronic machinery. The ontological and epistemological status of this independence has been discussed under terms such as truth and objectivity.

Independent of the legitimacy of the perspective outlined in the last paragraph, there are good reasons to study mathematics as a dehumanised practice. Firstly, the possibility and reality of mathematics as a dehumanised practice is a psychological and sociological phenomenon which deserved attention. In what ways does mathematics achieve dehumanisation? How is such a practice even possible? Why would humans want to engage in it? All these questions focus on *processes* of dehumanisation. Secondly, the dehumanised practices of mathematics result in a dehumanised handling of the issues mathematics is applied to. This is how the prosperity of a dehumanised practice such as mathematics effects culture as a whole. This perspective proposes to focus our discussion also on *consequences* of dehumanisation. All these discussions have the potential to deepen our understanding of mathematics from a sociological, psychological, and philosophical perspective.

Admittedly, one might want to add that there should also be a focus on the *ideology* of dehumanisation, which plays a central role when the values of objectivity and truth are wrongfully projected from 'pure' mathematics to applications of mathematics. Especially the belief that the choice of mathematical models in application is not arbitrary but objectively necessary turns mathematics into a questionable tool of power. However, as intriguing as such a perspective is, much has already been written about it (Davis & Hersh, 1980; Desrosières, 1993; Dowling, 1998; Porter, 1996; Skovsmose, 1994; Ullmann, 2008), and it is not the focus of this chapter.

Some readers might be uneasy with my description of mathematics as a dehumanising practice. Has the turn from mathematics-as-a-product to mathematics-as-a-practice (stimulated, e.g., by Pólya, 1945) not been a major step forward in the philosophy of mathematics, allowing for sociological perspectives on the human side of doing mathematics? Is the description of mathematics as a dehumanising practice not a step back to rightfully outmoded perspectives on mathematics? I argue otherwise, and for that I want to return to Weber's discussion of bureaucracy once more. What did Weber mean when he argued that bureaucracy works the better 'the more completely it succeeds in eliminating from official business love, hatred, and all purely personal, irrational, and emotional elements which escape calculation'? Note that Weber did not say that bureaucracy *was* the practice that realised these attributes! Of course, even bureaucratic administration leaves open doors for some degree of personal variation based, for example, on annoyance or compassion.[3] Dehumanisation can instead be understood as an *ideal* of bureaucratic practice. Bureaucratic administration may never be fully dehumanised, but it is conceived the better, the more it reaches this ideal. Nevertheless, it is clear that human beings execute administration, and research could come to analyse the rather diverse practices that exist within bureaucratic institutions. In the same sense, dehumanisation can be understood as an ideal of research and school mathematics today without saying that

3 Examples might be: receiving petitioners after closing time, allowing petitioners to hand in attachments for applications after their deadlines, offering superficial or profound consulting, providing tips how to get the most out of a specific situation.

research and school mathematics is not still human-made or that the mathematical practices we engage in are no worthy part of the study of what we call mathematics.

What then is this 'mathematics' that is asserted to be a project of dehumanisation? I do not wish to attempt to answer the highly controversial question what mathematics *is*. Instead, I propose to further discuss two very general mathematical practices which have, beyond any doubt, become paradigmatic for the question what mathematics might be, namely *calculation* and *proof*. Calculation refers to a more hands-on characteristic of mathematics. It is closely connected to applications of mathematics and can be traced back to the very beginnings of human civilisation. Proof, then, at least in the Western tradition, refers to a more philosophical approach towards mathematics with only indirect connections to applications. It is closely connected to the manifestation of mathematics as part of academics and can be traced back to Ancient Greece.[4] The following parts of the chapter will therefore be dedicated to the discussion in how far calculation and proof can be understood as a project of dehumanisation.[5]

Dehumanisation through calculation

Calculation is probably the central driving force of dehumanisation through mathematics. First, calculation owes its efficiency to a certain disregard of the objects of investigation, thus, when these objects are people, opening a space for the dehumanisation of others. Second, calculation demands from its applier a certain mindset that is not unlike the dehumanised mindset of the bureaucrat. Consequently, calculation appears to be a worthy start for a discussion of dehumanisation through mathematics.

Calculation, here, should be understood in a wide sense as any manipulative practice within a calculus and any application of such

4 Note that there are traditions of the justification of mathematical knowledge that are closely connected to calculation and application, for example in Ancient Chinese mathematics (Chemla, 2012).

5 Some readers might notice that I have already discussed these issues elsewhere (e.g., Kollosche, 2014), but while my earlier approach towards that topic had been guided by socio-economic and didactical perspectives, I want to dare a closer look at epistemological aspects of mathematics here.

calculi. 'Calculus', in the English language often closely associated with the infinitesimal calculus, refers to a system comprising a set of allowed signs, rules for their combination to statements, and rules for the manipulation of such statements (Krämer, 1998, p. 29). Besides infinitesimal calculus and among others, we have arithmetic, algebraic, and propositional calculi. The following discussions will circle around arithmetic and elementary algebra, but the points made are meant to be valid for any form of calculation.

Calculation as a shared practice

For reasons of administration, every highly organised state appears to develop some proficiency in calculation. In the Rhind Papyrus, one of the oldest surviving textbooks for mathematics, dating back to Ancient Egypt around 1550 BC, we find the following mathematical problem:

> A quantity, ¼ added to it, becomes it: 15.
> Operate on 4; make thou ¼ of them, namely, 1; the total is 5.
> Operate on 5 for finding of 15. There become 3.
> Multiply: 3 times 4. There become 12.
> The quantity is 12, ¼ of it is 3, the total is 15. (Chace et al., 1929, Vol. 2, Plate 48)[6]

We can see that the problem is posed without any contextualisation. Later examples in that script for solving linear equations feature measures for volumes without any change in the calculative techniques. It can be assumed that those who worked with this textbook were ready to apply this kind of calculative practice to a variety of situations. Or, to put a different emphasis on that last statement: a variety of situations came to be dealt with using identical mathematical techniques. The papyrus also includes many distribution problems, in which usually bread loaves are distributed among men, whereas in one problem measures of beer are distributed without changing the calculative techniques, except for measure conversions in some cases. By contrast, it is fair to doubt that bread loaves and measures of beer were the most pressing problems of distribution for the Egyptian administration. Calculation seems not to depend on what the numbers stand for, be it abstract entities such

6 Chace et al. (1929) provide literal translations, hence the bumpy expression.

as measured quantities and distribution shares or rather real and alive objects such as flocks of animals. For the practice of calculation, context is irrelevant.

Now, imagine the problem presented above had a context, featuring, for example, a shepherd who lent his flock for ¼ interest and was paid back a flock of 15 animals. The question how many animals he lent in the first place could perfectly well be computed by the Egyptian solution. But what would '5' in the second line or '3' in the third line of the cited problem actually mean in our context? We cannot tell and we do not need to care. Calculative techniques work in a mechanical way, irrespective of context, which means that they can be applied also to human affairs without any regard for individual concerns of the objects they are applied to. They bear the possibility of dehumanisation.

In his *Remarks on the Foundations of Mathematics*, Ludwig Wittgenstein (1978) reflected on the nature of rule-following in mathematics. He sees this phenomenon based not only on the experience of shared perception, but also on our experience that imitation (in the sense of copying somebody's actions and obtaining the same result) is possible. For Wittgenstein, the possibility of this kind of conformity is a basic truth of our experience and the beginning of any explanation. Following rules is only possible in the areas of our experience which allow for such conformity. The very idea of a 'rule' results from this experience of repetition of perception and act. Verbalised rules then are first and foremost descriptions of repeating perceptions and actions. Only on this basis can they be understood as prescriptive (in the sense of prescribing perceptions and actions whose description would be that very rule). This line of thought shows, as Wittgenstein stressed, that following rules is a cultural achievement and specifically human. Nevertheless, the very nature of following rules is not to be oneself but to follow the other, to universalise perception and action, to ignore the peculiarities one might experience, to eventually dehumanise the processing of our affairs.

The annulment of meaning

Already the prehistorical example from Ancient Egypt teaches us that calculation is a technique whose internal working is ignorant to what it processes. This ignorance is one source for dehumanisation through calculation, for calculation serves all its objects equally – be they pebbles, bread loaves, sheep, or human beings. Already here, calculation means following specific rules for perceiving and acting with numbers.

However, the history of formalisation teaches us that ignorance can have limitations. For example, the disputes accompanying the introduction of the number zero and of infinitesimals were primarily based on the question what these entities ought to be (Kleiner, 2001; Krämer, 1988a). We might not be able to say what '5' means in the second line of the problem discussed in the last section, but at least we can refer to perceptions where we see five of something, which is a strategy for manifestation that we cannot use for zero and infinitesimals. Calculating with zero and infinitesimals actually *requires* to ignore the question what they might mean in reality. Apparently, not everybody was willing to make this sacrifice. Indeed, we can see that such instances of the ignorance of the question where concepts of mathematics are to be found in our world is a matter of modern times. When trying to locate where this attitude towards meaning has changed, we suddenly encounter developments that go far beyond mathematics and will prove relevant in a variety of ways.

In *The Order of Things*, subtitled *An Archaeology of the Human Sciences*, Michel Foucault (2007) tracked changes in 'the epistemological field [...] in which knowledge, envisaged apart from all criteria having reference to its rational value or to its objective forms, grounds its positivity' (p. xxiii). With central importance to this project, he discusses changes in the use of signs in different cultural arenas. Foucault's central finding is that at the beginning of the seventeenth century, signs were no longer assumed to be inseparably connected to the signified as they had been conceived ever before. Suddenly, they became to be considered arbitrary human constructions. In the arena of literature, he analyses the monumental novel *Don Quixote* by Miguel de Cervantes (1605), in which the protagonist imagines adventures as a knight only to be cast back to his profane reality. Foucault (2007) concluded that

> writing has ceased to be the prose of the world; resemblances and signs have dissolved their former alliance; similitudes have become deceptive and verge upon the visionary or madness; things still remain stubbornly within their ironic identity: they are no longer anything but what they are; words wander off on their own, without content, without resemblance

> to fill their emptiness; they are no longer the marks of things [...]. The written word and things no longer resemble one another. (p. 53)

In the arena of the price for material goods, prices were no longer inseparably linked to the expenses of production, storage, and distribution but were legitimate to take any value. In the arena of medicine, the idea that plants sensuously resembled the body parts or infestations they were able to cure was replaced by a logic of empirical enquiry guided by measurement and logical order. In general, the idea of resemblance was replaced by the idea of an ordered choice of signs. What such an order could and should look like is not an essentialist question; it is a question of logic. In consequence, the seventeenth century sees the rise of a vivid academic discourse on how concepts should be framed and related.

Algebra is no arena of Foucault's (2007) study, but it could just as well have been. Variables are a special sort of mathematical signs, and they change their nature within the time frame discussed by Foucault. Sybille Krämer presented an intriguing study of the change of the use of operative signs in mathematics.[7] Already in the Ancient Egyptian problem discussed above, we see the appearance of variables, there translated as 'a quantity' (elsewhere as 'a heap'), but noted as only one sign in the original hieroglyph script. Krämer (1988a) noted that this use of variables follows the idea that it stands *in place of* a well-defined and yet-unknown number, which eventually can be computed. Variables, here, have a very specific meaning.[8] They represent distinct numbers. Euclid's geometry of Ancient Greece did not include variables for numbers, but it used line segments as general entities irrespective of their actual lengths. In this sense, the abstract line segment can be understood as a variable for lines segments with specific lengths. In some cases, the specific length will follow with necessity from other data in a geometrical construction, while in other cases, the general lines segment is allowed to assume any length. In the latter sense, the line segment can be understood as a geometric variable, as it no longer

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7 Krämer (1988a) and Krämer (1991) are two rich and original studies in German. Where possible, I will refer to Krämer (1988b), which is an early summary in English.

8 The epistemology of the term 'variable' from Lat. *variabilis*, meaning 'changeable', is misleading in this case. Some scholars speak of 'apparent variable' or 'bound variable' when they refer to variables in the function of a placeholder.

stands in place of a well-defined and yet-unknown length. It took a long international development to introduce the idea of the variable to algebra as well, including a lot of work by scholars from the Islamic world. In Europe, the popularisation of use of variables in mathematics is often attributed to François Viète's *Isagoge in Artem Analyticam* from 1591, where variables are used to represent a wide range of numbers as in many of today's equations such as $a + b = b + a$ or $y = 7x - 2$. Krämer (1988b) analysed:

> Thus it becomes possible to formulate rules of algebra with universal validity. Unlike the ciphers, the letters are no longer signs for single numbers, but rather signs for the whole class of numbers satisfying a given equation by substitution. The rules which refer to the transformation of equations can in this way be written down in a formal language. This means that their validity is independent of the numerical values entering into the calculation. Algebra becomes the transformation of series of signs according to rules which have no relation to the meaning of the signs. (p. 182)

We could also say that the meaning of Viète's signs is defined by their use in the calculus, the formal language of mathematics, alone, and not by any reference to a meaning beyond them. This breakthrough laid the foundations for many influential developments to come, all depending on the use of signs as ontologically independent entities. René Descartes established an analogy between algebra and geometry and thus opened geometry up for calculation as a tool for solving problems. Isaac Newton and Gottfried Wilhelm Leibniz developed an infinitesimal calculus, in which infinitesimals as well as functions become entities of calculation. Leibniz already worked on a logical calculus and developed the idea that all the truths of the world could be computed on the basis of a sufficiently developed formal language. Here, truth becomes a question of the logic of signs, which no longer represent anything.

The cultural impact of calculation

Now, if thought indeed changed from a logic of resemblance towards a logic in which signs were set loose, would mathematics be a leader or a follower in this process? Viète published his algebra in 1591, whereas *Don Quixote* was published in 1605. But this difference might be misleading, as it can be assumed that in all social arenas, the change came slowly and was only catalysed by intellectual pioneers, whose appearance in time is somewhat random. Foucault (2007) stated that the change occurred 'roughly half-way through the seventeenth century' (p. xxiv) and that the logic of resemblance was in use still 'at the end of the sixteenth century, and even in the early seventeenth century' (p. 19). Note that Descartes's analytical geometry, the next big step in the history of mathematics and in the use of signs as independent entities, was not published before 1637 and was received with astonishment even then!

Still, there are other reasons to assume that mathematics was not a mere follower in this transition in the use of signs. Calculation techniques as in the Egyptian problem above had already illustrated that the manipulation of signs is possible without any reference to their meaning. The signs themselves were required to resemble something real, but their handling was not. The geometry of Euclid had already made implicit use of the idea of variables, albeit restricted to geometrical contexts. In the third century AD, Diophantus of Alexandria had introduced ancient Asian techniques for adding lengths and areas without the scruples of the earlier Ancient Greek tradition, as did many Persian and Arab scholars in the middle ages. Fifteenth-century Europe also saw the introduction of the Indian positional notation system (popularised through economic applications), which brought with it a further appreciation for the efficiency of sign manipulation as in the algorithms of written calculation and a raising tolerance for meaningless signs such as the zero. These preconditions and developments have made it easier for Viète and those who followed to take the next step. So, mathematics seems to have been *a*, if not *the*, protagonist in the culture-wide change of the understanding of signs (Krämer, 1991).

Problematising dehumanisation through calculation

While calculation practices which are ignorant of the meaning of its manipulative steps have flourished for more than 3000 years, modern mathematics refuses to ask for the meaning of the values and expressions of calculation altogether. Roland Fischer (2006) pointed out that this ignorance is a virtue: mathematics would not be useful for practical affairs, if it was compelled to explain the meaning of every concept and manipulation. It is useful exactly because it can rely on calculation alone. All that may be so, but this potential of calculation comes with a price. The disregard for meaning requires those performing calculations to deny their individual thoughts in calculation and can operate on human beings only after reducing them to calculable magnitudes.

Formalism is the elaboration of the denial of meaning as an attitude in the philosophy of mathematics. It assumes mathematics to be nothing but a rule-based game with signs. The signs, representing and constituting mathematics, are held to have a meaning only within the game of mathematics. Reuben Hersh (1997) stressed that, from such a perspective, the applicability of mathematics cannot be explained; it must appear as an astonishing coincidence.[9] However, applicability requires explanation, for, as Gottlob Frege (1960) argued, 'it is applicability alone which elevates arithmetic from a game to the rank of a science' (p. 187).

By abandoning the logic of resemblance, mathematics was able to set loose the power of its formalistic apparatus, but it lost a dimension of self-reflection. No longer asking for more than formal explanations of what a mathematical concept stands for, what a mathematical proposition says, what a mathematical procedure does, means losing the ability to critically reflect on our use of mathematics. Of course, mathematics is still widely applied in our societies, but the question if these applications are justified, the question in how far the mathematical model actually resembles our worldly problem, is no longer a matter of mathematics.

The dialectics of the use of calculation for the processing of social affairs were best described by the Frankfurt School in philosophy. Max Horkheimer (2004) argued:

> As soon as a thought or a word becomes a tool, one can dispense with actually 'thinking' it, that is, with going through the logical acts involved in verbal formulation of it. As has been pointed out, often and correctly, the advantage of mathematics – the model of all neo-positivistic thinking – lies in just this 'intellectual economy.' Complicated logical operations are carried out without actual performance of all the intellectual acts upon which the mathematical and logical symbols

9 And mathematicians *are* astonished: Check, for example, Eugene Wigner's (1960) infamous paper on 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences'.

> are based. Such mechanization is indeed essential to the expansion of industry; but if it becomes the characteristic feature of minds, if reason itself is instrumentalized, it takes on a kind of materiality and blindness, becomes a fetish, a magic entity that is accepted rather than intellectually experienced. (p. 16)

A central line of critique of calculation as a social practice questions the legitimacy and effects of a practice which has to atomise its perception of the world into countable entities. Horkheimer and Adorno (2002) explained:

> Bourgeois society is ruled by equivalence. It makes dissimilar things comparable by reducing them to abstract quantities. For the Enlightenment, anything which cannot be resolved into numbers, and ultimately into one, is illusion; modern positivism consigns it to poetry. Unity remains the watchword from Parmenides to Russell. All gods and qualities must be destroyed. (pp. 4–5)

They go further in proposing that the mathematical handling of affairs is actually rendering our perception of reality and blocking aspects which cannot be dissolved into patterns of sameness and repetition:

> When in mathematics the unknown becomes the unknown quantity in an equation, it is made into something long familiar before any value has been assigned. Nature, before and after quantum theory, is what can be registered mathematically; even what cannot be assimilated, the insoluble and irrational, is fenced in by mathematical theorems. (p. 18)

These practices do of course leave an imprint both on people who are subjected to them and on people who are performing them:

> Not only is domination paid for with the estrangement of human beings from the dominated objects, but the relationships of human beings, including the relationship of individuals to themselves, have themselves been bewitched by the objectification of mind. Individuals shrink to the nodal points of conventional reactions and the modes of operation objectively expected of them. (p. 21)

Concerning the conduct of the self when calculating, Wittgenstein (1978) demonstrated that calculation rests on rules. For Wittgenstein, rules do not hold any inner truth, they are nothing but patterns of repeated action. The whole sense of rules is securing that people can agree on procedures that yield the same results irrelevant of who is performing them. Such predictability lies at the basis of games such as chess, and we would just as well expect it from calculation. It follows that learning to calculate includes learning to follow pre-given rules, to perform acts that are not original to the self but copied from others. This is how calculation dehumanises the self.

In mathematics education research, there remains a strange lack of reflection on how learners cope with the dehumanising side of calculation. Renate Voswinkel (1998), a pastor reflecting on her troubles with mathematics in school, reported:

> We learned our times tables. I memorized the rows and only then checked if it was right that 7 times 3 is 21. 7 times 8 is 56, that is what I told myself in the morning when washing before we would write a test on the tables of eight. I did not keep this in mind because I kept thinking about further-reaching things that I cannot remember today. The result was an increasing quiet devaluation of my own thoughts. I forbade myself to think, because it confused me, although I made my own connections in all other subjects, had ideas, developed a lot of imagination [...]. (p. 18, my translation)

Concerning the discourse on others, there are plenty of examples of how calculation is instrumental in dehumanising people. Andreas Bell (2016) discussed how to allocate donor organs, where the usual practice in Germany relies on a mathematical model that calculates the individual claim on a donor organ on the basis of a few personal variables. Bell argues that although a society has an interest in installing a mathematical mechanism for the transparent allocation of donor organs on the basis of principles such as the social optimisation of this allocation, no mechanism can satisfy all possible expectations concerning optimisation, equity, and compassion. Here, reducing individual cases to a pre-defined set of variables is necessary in order to apply any systematic form of decision-making at all, and yet this process necessarily leads to a dehumanisation of those waiting for a donor organ.

Let me also cite an example from Philip Davis' (2012) epilogue to a recent study edition of *The Mathematical Experience*:

> We are indeed living in an increasingly techno-mathematized world. A recent hospitalization for a minor complaint drove this home to me. I was subjected to a battery of tests carried out on a variety of devices each of

> which produced either numbers or a waveform. The medical attendant marked down all the numbers and perhaps a fast Fourier transform was applied to the waveform to obtain more numbers. As a patient, I was transfigured – some might say dehumanized – into a multicomponented vector. (p. 491)

My last example is an extreme case but rather revealing in its simplicity. Look at the following problem from a mathematics textbook in Nazi Germany:

> In 1936, the annual expenditure for

1)	33 770	welfare children	19 881 000 RM[10]
2)	131 942	insane and mentally deficient	94 636 600 RM
3)	238 094	hereditary defective (deaf-mutes etc.)	166 000 000 RM

> Calculate the cost per head [...].
>
> How many single-family houses at 5000 RM could be built with the sum required for the insane (or the hereditary defective)?
>
> How many families could make their living from these sums (1500 RM per year)?
>
> (Frank, 1939, p. 38, cited in Kütting, 2012, p. 11, my translation)

The scandal here is that the legitimacy of care for people in need is reduced to only one variable of their existence, namely to what they cost society. The comparisons that the demanded calculations suggest, though they are economic nonsense, were meant to raise the acceptance for the euthanasia policy of Nazi Germany. While this example is extreme, we will find similarly ambiguous uses of calculatory practices as legitimisations throughout today's public life (Porter, 1996). The dehumanisation of human beings through mathematics is not a sporadic accident of the application of calculation, it is its predominant mode of operation, as has long been proposed by Davis and Hersh (1986):

> The final intent of the application of mathematics to people is to be able to compare two individuals or groups of individuals, to be able to arrive at a precise and definitive opinion as to which is taller, smarter, richer, healthier, happier, more prolific, which is entitled to more goods and more prestige, and ultimately, when this weapon of thought is pushed to its logical limits and cruelly turned around, which is the most useless and hence the most disposable. Whenever anyone writes down an

10 RM stands for *Reichsmark*, the official currency of Germany at that time.

> equation that explicitly or implicitly alludes to an individual or a group of individuals, whether this be in economics, sociology, psychology, medicine, politics, demography, or military affairs, the possibility of dehumanization exits. [...] What is not often pointed out is that this dehumanization is intrinsic to the fundamental intellectual processes that are inherent in mathematics. (p. 283)

Dehumanisation through logic

The relationship between mathematics and logic can be heavily debated. There had been the ambitious but failed attempt by Whitehead and Russell to ground all mathematics on formal logic (George & Velleman, 2002). Others might argue that mathematical work can be disturbingly illogical, only to return to logical forms after a rather wild process of exploring and conjecturing. One way or the other, the *product* of mathematical work will be a theory, which is expected to follow certain criteria of logic, for example that it does not allow to deduce within it two mutually contradictory statements. In the *process* of creating such products, mathematicians will, to some extent or the other, use logical thinking. Eventually, even school mathematics, usually mirroring more elaborated mathematical theories in simpler forms, is a logically organised product.

A common assumption is that logical thinking is an innate capacity of human beings and that self-discipline and good education allow the individual to exploit this capacity to the fullest. From this point of view, logical thinking could be said to be a central part of evolving one's humanity. Psychologically, that may be a way to see it, but sociology casts doubt. Is logical thinking really an innate capacity? Valerie Walkerdine (1988) radically criticised traditional psychology and showed in many experiments that what we call rationality is actually a form of the conduct of the self that is learnt in social interaction. But if we, following this insight, begin to understand logical thinking as a cultural phenomenon, it appears to be astonishing that, using logical thinking, different people come to the same conclusions, find the same arguments compelling, see the same contradictions. Here, I will explain this particularity by demonstrating that logic is a dehumanised practice in that it offers a mechanism of thought which negates individual concerns.

Fundamentals, and how (not) to read them

There is no single answer to the question what logic is. *Formal logic* is highly mathematical, providing different calculi in which statements can be noted and manipulated through computation. While formal logic is a modern phenomenon, we find an abstract and, to some extent, already formalised approach in Aristotle's discussion of certain and uncertain *forms of inference* (Aristotle, trans. 1989). Such descriptions of logical thinking are, in modernised form, of interest for psychology, which studies individual capacities to perform such forms of thought. However, even Aristotle's approach can be argued to rest on some epistemological assumptions (as argued, e.g., by Leibniz, 1765/1896, pp. 404–410), which were already, though not systematically, mentioned by Aristotle, and systematically discussed by scholars such as Arthur Schopenhauer (1903). While categorising forms of inference and discussing logical calculi is rather technical work, the underlying assumptions are very far-reaching decisions of how to think about our world. My analysis of dehumanisation through logic will begin here.

This perspective will follow a somewhat Eurocentric interpretation of what logic might be. Notwithstanding the fact that other cultures developed reflections on logic or even described other forms of reasoning as logical, there are good reasons for the focus on Ancient Greek philosophy: first, it provides us with very early sources on the philosophy of logic, which allows a far-reaching look into the history of such reflections. Second, Ancient Greek logic has been studied by many scholars, upon which we can rely here. Third, Ancient Greek logic has been highly influential for European and modern philosophy and mathematics. However, it should be noted that there was no monolithic 'Ancient Greek logic', that the subject itself was much debated at that time, and that the philosophical worship of Ancient Greek logic as it has been perceived and retold by philosophical tradition may have clouded our view on epistemological alternatives. In any case, it should be noted that when I write 'logic' I refer to the Eurocentric reception of Ancient Greek logic. This use of the word is not meant to deny the existence and legitimacy of other forms of logic.

The formulation and meaning of these foundational assumptions of logic are a matter of ongoing debate, so that any presentation is already biased by a specific interpretation. Allow me to present the assumptions, which are canonically called ‘principles’ or ‘laws’, in an interpretation following Klaus Heinrich (1981), only to provide more diverse context later:

1. Things stay the same; they do not change. (Law of identity)
2. Everything is or is not; there is no other way. (Law of excluded middle)
3. Nothing both is and is not. (Law of excluded contradiction)
4. Everything has a reason and is defined by it. (Law of sufficient reason)

Even though these words may provoke many associations, their meaning appears not to be straightforwardly clear. I do not wish to summarise the vast landscape of interpretational controversies here, but let me give some short examples for the interpretation of the law of identity. In the years around 1700, Leibniz (1896) held the law of identity to say that ‘everything is what it is’ (p. 404), or, more formally, that ‘A is A’ (p. 405), thus reducing the law of identity to a mere tautology. Leibniz assumes that such ‘primate truths of reason [...] seem only to repeat the same thing without giving us any information’ (p. 404). He cautiously added the word ‘seem’ because he saw a function of the law of identity for the manipulation of formal logical statements, but he did not see in the law of identity anything more than a self-evident statement. But would the law of identity have fascinated philosophers over centuries if it was a mere tautology, if it was not ‘giving us any information’?

Foucault sets the scene for a different perspective. In his study on insanity, Foucault (1954) showed that the idea of insanity came into being only in modern times, perceived as a threat to reasonable thinking and accompanied by asylums as new institutions and psychology as a new academic discipline. Apparently, as natural and indispensable as the idea of insanity may seem to us today, there had been a kind of thinking in which this idea played no role at all for understanding our world. Based on this insight, Foucault (1966) studied more general patterns of thinking and reasoning over time, and showed that they change severely, including the role of logico-mathematical perspectives of understanding. He called his approach *genealogy*. Foucault (1984) wanted to historically trace ideas not in order to show an inevitable way to any presumably necessary understanding we might have today, but in order to reveal the implicit meaning of the ideas by an analysis of alternatives they were positioned against, of fears, needs and desires that promoted their development. Might logic be a cultural answer to a specific configuration of fears, needs, and desires?

Attend to the following passage where Aristotle (trans. 1933) touched on the problem of identity:

> Thus in the first place it is obvious that this at any rate is true: that the term 'to be' or 'not to be' has a definite meaning; so that not everything can be 'so and not so.' Again, if 'man' has one meaning, let this be 'two-footed animal.' [...] If on the other hand it be said that 'man' has an infinite number of meanings, obviously there can be no discourse; for not to have one meaning is to have no meaning, and if words have no meaning there is an end of discourse with others, and even, strictly speaking, with oneself [...]. (1006a–b)

Here, identity appears to be a matter of the fixation of meaning in a social discourse. This perspective suddenly positions logic in the social realm. Identity demands that the meaning of concepts is made independent from individual interpretation, or, in other words, that their meaning becomes dehumanised. But why did Aristotle have to argue for the law of identity in the first place? Were there any alternatives whose legitimacy Aristotle wanted to disprove? What then is the historical background on the basis of which we can explicate the meaning of the logical assumptions listed above? It may seem that we need a genealogy of the very foundations of logic.

Genealogy of logic

Jean-Pierre Vernant's (1982) *Origins of Greek Thought* provides an intriguing account that logic is not inherent but a cultural phenomenon that can, in the Western tradition, be traced back to Ancient Greece. I owe most of the philosophical perspective on this development to Heinrich (1981), whose research took as its objects of study 'the supressed of philosophy, and not the accidentally suppressed but that, which in the systems of thought, in the rationalised systems of occidental thinking, indeed returns' in the form of a compulsive and unconscious formation of thought (p. 173, my translation).

Hesiod's *Theogony* and Homer's epics illustrate the understanding of the world in Ancient Greece in the eighth century BC. It was, as in many other cultures, based on a polytheistic religion. In this worldview, the worldly forces were humanised in the sense of a human-like representation as gods. For example, Ares stood for war, Demeter for agriculture, Dionysus for ecstasy, and Hermes for trade. Crisis in worldly affairs such as droughts, earthquakes, or diseases could be understood through the tempers of and struggles among the gods. Especially, the Greeks believed that all descendants of a god inherited his or her virtues and vices and could never escape this fate – a belief that we will return to.[11]

Historical changes led to doubt about the legitimacy of the polytheistic worldview (Vernant, 1962/1982). The vast trade network of Ancient Greece imported foreign religions, making the polytheistic worldview appear as a mere possibility among others. Wars led to the destruction of kingdoms whose legitimacy was closely connect to the old myth. Democratically organised city states (note that only the male aristocracy belonged to the *dēmos*) such as Athens developed a culture of public discussion where soon not only political but also moral and religious standpoints came to be questioned. Whether the myth was the appropriate way to explain the world became a pressing question. Philosophy developed within this intellectual crisis as the project of finding better explanations. In this context, Heinrich (1981) reported that Plato had Socrates mourn that 'it is the woe of the philosopher to be confused this way, for confusion indeed is the only source of philosophy'.[12]

11 Heinrich (1981, p. 99) cited the Curse of the House of Atreus as an illustration, which is documented in the eleventh song of Homer's *Odyssey*: The mythical god-king Tantalus, a son of Zeus, had offered his dismembered son Pelops as a meal to the gods to test their omniscience. They reassembled Pelops, revived him, and cursed the lineage of Tantalus. All descendants of Tantalus, including Pelops, his son Atreus, and his son Agamemnon, were subsequently involved in clan murders, hatred, and conspiracies. No descendant of Tantalus could escape this fate. The curse was inherited, and inheritance was so inescapable that even the gods could not exclude Pelops from the hereditary curse.

12 Heinrich (1981, p. 31) differs from usual German and English translations of the ambiguous Greek original. For example, Harold N. Fowler translated: 'For this

What follows, beginning in the sixth century BC, are philosophical attempts for a reliable theory of the world. Providing an overview of these attempts would carry us too far off, but I will return to the ideas of Anaximander of Miletus, probably a student of Thales, and of Parmenides of Elea, a student of Xenophanes, both living in that time period and laying the intellectual ground for the work of Socrates, Plato, and Aristotle.

Deduction

Looking for the origins for the concept of deduction, one ends up with Anaximander, who composed the philosophical poem 'On Nature' in the first half of the sixth century BC.[13] There, Anaximander (2007) argued that 'everything either is an origin or results from an origin' (p. 35, my translation).[14] Not much is recorded which would further qualify this thought. However, what can be said is that, with Anaximander, the idea was set loose that things do have a *reason*. Anaximander goes on philosophising about the final reason, which we will get back to. For now, it is important to say that Anaximander's worldview was the oldest surviving Greek view not to be built on divine entities. The reason for something to happen was not to be found in the realm of the gods but in nature.

Some scholars say that Anaximander founded physics as he was the first to propose a cosmology that worked without gods and asked for reasons. Aristotle's (trans. 1933) proposition 'that we must obtain

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feeling of wonder shows that you are a philosopher, since wonder is the only beginning of philosophy' (Plato, trans. 1921b, 155d). Heinrich argues that the Greek word πάθος (*páthos*) does not merely mean 'feeling' but has the connotation of suffering, and that θαυμάζειν (*thaumázein*) is not merely 'wonder' but something negative. I tried to provide an English translation in accordance with Heinrich's interpretation.

13 This poem has only survived through the citations of fragments of it by others. Gemelli Marciano (2007) compiled all the fragments available and offers a good translation into German. For the lack of a compilation with a translation into English, I will refer to the German compilation and offer translations from it.

14 The ambiguous Greek original ἀρχή (*archē*) can be translated to 'beginning', 'origin', 'sovereignty', 'sovereign' or 'principle'. Gemelli Marciano (2007) translates it to '*Prinzip*', which would be 'principle' in English. Following Heinrich (1981), I chose a different translation to emphasise the close connection of Anaximander's thought to deductive thinking.

knowledge of the primary causes, because it is when we think that we understand its primary cause that we claim to know each particular thing' (983a) shows that this idea set a standard even some centuries later. Through the Attic philosophers, the idea of deductive reasoning was set as a standard of Western academia.

It is interesting to note the structural analogy between inheritance and reason: just as the gods passed their traits to their offspring, which then could not escape this fate, deduction presupposes that concepts necessarily hold all the properties of the concepts from which they derive. It is more than a bizarre side note that the move from myth to physics appears to be merely a replacement of gods with natural forces while keeping the overall architecture of argumentation untouched.

Identity

As I have argued earlier, understanding identity as the tautology that 'a is a' does not transport any meaning, needless to say. Instead, the principle of identity should be understood as the plea, the postulation, even the command that there *should* be things that stay the same. We can understand this postulation more socially following Aristotle who argued that people should make sure that they talk about the same things – from person to person and from instance to instance. We can also understand this postulation more religiously as the belief that our world is indeed based on things which do not change, and mathematics might be seen to belong to these things. Indeed, Socrates and Plato followed this essentialist belief, as did Anaximander and Parmenides.

Anaximander (2007) knew that in his cosmos of deductions, the deductive chain would need to start somewhere. He argued that 'there is no origin of the infinite, for otherwise it would be confined' (p. 35, my translation). He continued that this 'infinite' had 'not emerged', was 'imperishable', 'immortal', 'indestructible', 'eternal' and 'not aging', it 'seems to be the origin of all other things' (pp. 34–37, my translation). In the cycle of time, the world emerges from the infinite, only to perish to it again. The inevitability of Anaximander's infinite, on which everything depends, is more relentless than the mythical gods: at least, the latter had, through their humanesque character, a free will and could be fought. In Anaximander's cosmos, fate leaves no hope of being negotiable.

About half a century later, Parmenides (trans. 2009) composed his own poem 'On Nature', in which the author ascends to the gods to hear an epiphany from Dike, the goddess of justice, morals and fair judgement. Therein, Parmenides formulated a first version of the law of identity and introduced the concept of 'truth', mostly referred to as 'being', to the philosophical discussion (pp. 56–57). Parmenides explained that 'that Being is ingenerate and imperishable, entire, unique, unmoved and perfect' (p. 64), 'it never was nor will be, since it is now all together, one, indivisible' (p. 66) and it is 'bound fast by fate to be entire and changeless' (p. 76). The analogies to Anaximander's infinite are obvious. But while Anaximander's infinite is a necessary element of his explanation of the world, Parmenides' truth is an idea that belongs to a discussion of how to reason properly. Consequently, Parmenides has often been considered the founder of logic. It might be added as a side note that his poem also includes the oldest surviving example of a deduction.

What drove the intellectual development that resulted in the invention of truth? When we remember the state of confusion the Ancient Greek aristocracy suffered, the idea of truth offered too good a promise. Heinrich (1981) summarised this promise in the fictional wording: '"Fear not", for there is an existence which remains untouched by fate and death' (pp. 45–46, my translation). We find reassurance for such an interpretation in Parmenides' poem itself. Parmenides (trans. 2009) wrote that Dike did not allow truth 'either to come to be or to be perishing but holds it fast' (p. 68). Dike also asked Parmenides to stay away

> from that on which mortals with no understanding stray two-headed, for perplexity in their own breasts directs their mind astray and they are borne on deaf and blind alike in bewilderment, people without judgement, by whom this has been accepted as both being and not being, the same and not the same [...]. (p. 58)

While all beliefs include the danger of impermanence, truth would, by definition, never disappoint anyone. The price for that security is that truth is also completely independent from humans, that knowledge is dehumanised. In this vein, the vernacular expression of 'dead knowledge' for scientific truths resembles the ideas of Parmenides and his disciples rather well. Indeed, Plato (trans. 1921a) had the death-sentenced Socrates say ‘that those who pursue philosophy aright study nothing but dying and being dead’ and that ‘it would be absurd to be eager for nothing but this all their lives, and then to be troubled when that came for which they had all along been eagerly practicing’ (64a). There might be becoming and perishing in life, but nothing but eternal truth in death.

Dichotomies

The last quote from Parmenides (trans. 2009) also gives a hint that he already had an understanding of what earlier I presented as the laws of the excluded middle and the excluded contradiction. In fact, he is often attributed as the first philosopher to ever formulate these laws. In the quote above, the perplexed ‘two-headed’, who are incapable of judgement, unable to say what is, accept things ‘as both being and not being’, as both true and false, thus violating the law of the excluded contradiction (p. 58). Elsewhere, Parmenides added that ‘mortals’ suppose some things ‘to be coming to be and perishing, to be and not to be, and to change their place’ (p. 78). Here we have the connection in one line: allowing contradiction would invite the forces of becoming and perishing into philosophy, but these are deadly forces that change the face of the earth, that are unstable, and thus no foundation for any stable worldview.

Why start thinking *like this*?

We followed Vernant (1982) in maintaining that a driving force of philosophy in Ancient Greece was to build a more reliable fundament for understanding the world than the polytheistic myth had been able to. Apparently, the idea of truth promised the possibility of a secure understanding in its purest form. Nevertheless, it remains interesting to ask why scholars in Ancient Greece started to think like this, on the grounds of these fundamental assumptions of logic. We should hesitate to explain this development by assuming a logical order of the world or of the human mind, for that would mean that all scholars and cultures who did not follow the assumptions of logic discussed above have not reached the right access to our world or have not developed the right way of thinking. Such a position would make us a complicit of the superiority which logicians such as Parmenides assume for the kind of thinking they present, whereas in research, we should seek to obtain an unbiased distance to what we study. Therefore, it is necessary to ask why logic is organised in this peculiar way, if there would be no other possibilities to explain the world on the basis of a concept of truth.

Now, is there a socio-cultural explanation for why logic assumed the form it did? I know only one such explanation: namely that this form copies an order which was already being lived in the patriarchal society of Ancient Greece. It would have been difficult to come up with an order of thought out of the blue, but it should have been easier to come up with an order that is an abstraction of lived social organisation. The analogy between the fate of gods and the law of reason already gave us a glimpse of such a connection.

When I refer to patriarchy here, I refer to a very specific organisation of society which marks the beginning of Greek history. Pre-patriarchal societies know no fatherhood, no possession, no male superiority, and usually worshipped the holy mother who brought life into the world (Lerner, 1986). Patriarchal societies introduce the ideas of fatherhood, of male rule over women and offspring, of marriage, of property, and of inheriting.

Horkheimer and Adorno (2002) claim that 'the generality of the ideas developed by discursive logic, power [*die Herrschaft*] in the sphere of the concept, is built on the foundation of power in reality' (p. 10). What might they have meant? Throughout their treatise, Horkheimer and Adorno (2002) point to the changes that have come with the introduction of patriarchy but do not illuminate that connection further. Only later, Gerhard Schwarz (2007) demonstrated the analogy between patriarchal and military hierarchy, while Fischer (2001) identified a structural analogy between the patriarchal and the logical order. Imagine a typical visualisation of hierarchies, a root network starting in one point and branching out downwards. At the top, we see the patriarchal father, the military commander, or the most general concept respectively. Branching out, we see the sons of that father and again their sons and grandsons; we see the soldiers second highest in rank, followed by those third highest in rank; we see concepts which are gradually more specific in meaning, for example, the triangle and quadrilateral branch out from the broader category of polygon. The triangle, in turn, divides further into equilateral, isosceles, and scalene triangles. Now note that the logical assumptions described earlier are inscribed already in the historical configuration of the family and the military: the law of identity means that you stay who you are in that configuration, you cannot change your position, cannot become your father's father or your superior's superior.[15] The law of sufficient reason means that everybody has a father, everybody has a direct superior. Admittedly, that might not be true for the founder of a house or for the commander-in-chief, just as Anaximander admitted for his logic that at least one thing cannot have an origin. Finally, the either-or resulting from the laws of the excluded middle and the excluded contradiction means that in regard of any person in the respective orders, this person either is your father or your direct superior respectively, or he is not. It is not possible that somebody is neither your father nor not your father, nor is it possible that somebody is both your father and not your father.

I know of no arguments which would explain why these analogies between the patriarchal family, military organisation, and the assumptions of logic discussed above are necessary. Instead, these analogies are very peculiar. Note that pre-patriarchal societies had no concept of fatherhood at all, and some partisan military groups partly renounce formal ranks. Note also that this logic cannot work if mothers were meant to enter it in a position equal to fathers, or if the paternity of a child is in doubt (which therefore causes a major crisis in the patriarchal order). Ancient Greece had also seen different ontologies which assumed that nothing is fixed and everything is in flux, as expressed by Heraclitus of Ephesus (trans. 1979) who, in the sixth century BC, stated that 'one cannot step twice into the same river' (p. 53), a saying further escalated by Cratylus, whom Aristotle (trans. 1933) reported to have added 'that it cannot be done even once' (1010a). A contemporary example might be the struggles around the erosion of the either-or in the dichotomy of gender (see Chapter 19 in this volume).

The analogous form of these notably particular social systems demands an explanation. An explanation for this apparent coincidence

15 That should be clear for the family. For the military it should be noted that, in Ancient Greece, positions were assigned by birth right and perhaps by economic status, without there being any system of promotion into higher ranks.

would be that the patriarchal family served as a model for the military order and for the relationships between the gods (a system which we could have successfully included in the discussion of analogies above), and that the latter entered philosophical deliberations and was eventually secularised into the system of logic. Then, the assumptions of logic are not only not necessary, they also bear the imprint of a very specific form of social organisation, which legitimises disposing of somebody's life, regarding women and children as property, exchanging individuality against obedience and loyalty, and holding social positions fixed instead of allowing people to become what they want. Besides, that logic also provides the basis for introducing further hierarchies such as those of social classes and of ethnicities. All this proposes that logic developed out of social practices which are at the root of dehumanising people.

Problematising dehumanisation through logic

Despite the inability of logic to provide fallacy-free theories (see Chapter 4 in this volume), the logically organised discipline of mathematics has provided an astounding complexity of insights, which are used in countless applications in our world. From that point of view, the organisation of thought in analogy with the patriarchal social order can be called a success. Most of us would not want to live without the technological achievements of our time, which rely heavily on applications of mathematics and on logically order discourses. However, it has to be acknowledged that this success stands in a dialectical relationship with practices of dehumanisation.

If we look at logic as a social practice, we may ask: What does this practice entail? Following the above analysis, it entails thinking in permanent and universal concepts that are arranged in hierarchies and irreconcilable antagonisms, and assuming the properties of these concepts to necessarily follow from the properties of concepts that stand higher in the hierarchy. It should be acknowledged that this is a very particular form of organising thought and that not all discourses will follow this example. As one choice among many, logical thought will have a specific potential, a specific price to pray, and specific limitations, which altogether deserve critical attention.

Logic is apparently a tool for the dehumanisation of others. Aristotle (trans. 1989) explicitly stated that his discussion of logic followed the purpose of understanding

> what sorts of things one must look to when refuting or establishing, and how one must search for premises concerning whatever is proposed, in the case of any discipline whatever, and finally the route through which we may obtain the principles concerning each subject. (52b–53a)

Aristotle's philosophy of communication did not aim at mutual understanding and amicable compromise; it did not even foreground the discovery of truth. Instead, Aristotle presented logic as a tool of rhetoric dominance. Pointing to inconsistencies in the other's use of concepts, to violations of antagonistic concepts and to contradictions that the other's ideas might result in, are techniques to devalue somebody else's thoughts. Instead, deduction is the attempt to force the other to accept one's own argument. Wittgenstein (1978) reflected on the logical argument as a command directed at the other:

> In what sense is [the] logical argument a compulsion?—'After all you grant *this* and *this*; so you must also grant *this*!' That is the way of compelling someone. That is to say, one can in fact compel people to admit something in this way.—Just as one can e.g. compel someone to go over there by pointing over there with a bidding gesture of the hand. (p. 81)

But why would the other follow the command to organise the discourse logically?[16] Here, the Ancient Greek philosophers fail to provide good reasons and turn to defamation instead. While Aristotle (trans. 1933) merely stated that those questioning logic 'lack education' (1006a), Parmenides (trans. 2009) scolded to keep back from the way

> on which mortals with no understanding stray two-headed, for perplexity in their own breasts directs their mind astray and they are borne on deaf and blind alike in bewilderment, people without judgement, by whom this has been accepted as both being and not being the same and not the same, and for all of whom their journey turns backwards again. (p. 58)

We see that the birth of logic was accompanied by a clear dehumanisation of others who think differently. Whether they still 'lack education' or

16 This is a question that Wittgenstein (1978) was puzzled by.

whether they suffer from mental illness and sensory disabilities, they always lack something to their full humanness which the protagonists of logical thinking do not lack. This is the technique that labels non-logical thought worthless and its wielder voiceless.

Logic is not only dehumanising others by stigmatising non-logical thinking, it itself leaves no possibility for people to express their individuality. Logic is not looking for the always different in the individual, it is looking for that which cannot change. Aristotle (trans. 1934) concluded that scholars following logic

> conceive that a thing which we know scientifically cannot vary; when a thing that can vary is beyond the range of our observation, we do not know whether it exists or not. An object of Scientific Knowledge, therefore, exists of necessity. It is therefore eternal, for everything existing of absolute necessity is eternal; and what is eternal does not come into existence or perish. Again, it is held that all Scientific Knowledge can be communicated by teaching, and that what is scientifically known must be learnt. (1139b)

In his reading of Anaximander, Friedrich Nietzsche (1962) tried to understand the mental state of the scholar. Anaximander's contribution for the appreciation of truth over the dynamics of becoming and perishing can hardly be overestimated. In fact, Anaximander regarded the eternal as the only legitimate existence and, as Nietzsche (1962) formulated, 'all coming-to-be as though it were an illegitimate emancipation from eternal being, a wrong for which destruction is the only penance' (p. 46). The totalising worship of logic bears the danger of devaluating life itself.

Concerning the self, logical thinking, although it might come with the promise of aligning with the eternal (Heinrich, 1981), demands a strict conduct of thought. I know of no psychoanalysis of this conduct of the self,[17] but I find Elizabeth de Freitas' (2008) report of Agnes, a fictional learner indulging in mathematics, to be a good provocation for scholarship:

17 Note the following comment by Paul Ernest (2016): 'I do not ask the interesting psychological question as to why persons might feel uncomfortable with uncertainty and have or feel the need for certainty or indeed of the place of uncertainty in the human condition. This would take me in another direction, possibly needing psychoanalytic theory, beyond the scope of my present inquiry' (p. 380).

> The peacefulness of deduction, the lack of dissent or debate, allowed for austere moments of meditation. Agnes indulged in that quiet hard work. She developed a passionate attachment to the symbolic world of mathematics. She saw beauty in mathematics. But the beauty captured in a mathematical proof was a purist's beauty that despised the messiness of the world. Agnes embraced this purist beauty and this method so completely that it crippled her will. She became possessed by reason; her body, emotions, and actions inscripted by logic. What began as tolerance and respect for the truth, devolved into a defensive self-abnegating disposition, a retreat from risk and adventure. An erasure of voice. (pp. 284–285)

De Freitas might have thought about a point made by Horkheimer and Adorno (2002) of how logic related to determinism and thus to the negation of choice:

> The arid wisdom which acknowledges nothing new under the sun, because all the pieces in the meaningless game have been played out, all the great thoughts have been thought, all possible discoveries can be construed in advance, and human beings are defined by self-preservation through adaptation – this barren wisdom merely reproduces the fantastic doctrine it rejects: the sanction of fate which, through retribution, incessantly reinstates what always was. Whatever might be different is made the same. (p. 8)

Consequently, we might argue that logical thinkers deny their roles in changing the world, that they silence their voices, that they confine themselves to discover and proclaim the eternal truths based on logic. This is a way of denying one's own humanity and reducing one's own intellect to what, to an increasing extent, even computers can achieve.[18]

Rehumanising mathematics?

I tried to show that mathematics, through its practices of calculation and logic, aims at a dehumanisation of action and thought. Admittedly, we might ask if other scientific disciplines do not seek a dehumanisation of action and thought themselves, if dehumanisation is not intrinsic to the idea of science producing objective knowledge. If we followed Aristotle's

18 See Chapter 2 for a note on computers proving or refuting mathematical theorems on the basis of logical calculations.

(trans. 1934) idea of science cited above, that would be so. However, science is no thoroughly logical enterprise, as already the co-existence of mutually conflicting theories in physics proves. Disciplines other than mathematics work empirically and have to offer theories that somehow work in practice. In contrast, mathematics, especially but not exclusively in its formalist fashion, reserved the luxury of considering itself a merely intellectual discipline. This ideal explains why mathematics can reject any empiricism, handle its objects in its liking, and mould them in forms that implement the idea of dehumanised action and thought like no other discipline. It is no coincidence that Leibniz (1996), who dreamt of a 'universal characteristic' that could express every scientific question and solve it through computation, was a mathematician.

Post-structuralism has taught us that the meaning of concepts is never fixed but in a state of permanent renegotiation. I guess that this was what Ole Skovsmose (2011) had in mind when he presented 'mathematics education as being undetermined', 'without "essence"', able to 'be acted out in many different ways and come to serve a grand variety of social, political, and economic functions and interests' (p. 2). The same should hold true for mathematics. Mathematics is not imposed on us but what we make of it. Can we alter the *modus operandi* of mathematics so that dehumanisation leaves the equation?

Attempts to present mathematics as a social practice are important, but not sufficient, steps in this direction. Indeed, the mathematical philosophies behind the works of scholars such as George Pólya (1945) and Imre Lakatos (1976) as well as attempts to write a philosophy of mathematics as a social practice as proposed by Davis and Hersh (1980) can be understood as projects to show the human side of producing mathematics. Obviously, this action is not logical in nature, but full of individual ideas, emotions, and conflict. These attempts in mathematics related closely to programs in mathematics education, which lay emphasis on activities such as problem solving and modelling instead of presenting mathematics in its logical structure or as a toolbox of calculative techniques. Although these perspectives help us to understand the doing and learning of mathematics, they do not reject the idea that the final product of all this activity is a logically ordered discourse that provides techniques for calculation. None of these perspectives question, for example, the legitimacy of the ignorance of meaning inscribed in calculative practices or the epistemological consequences of a two-valued logic.

Attempts to alter the inner working of mathematics itself are rare. A few general ideas can be found in feminist perspectives on mathematics and its education (see Chapter 19 in this volume), and a new perspective has secured recent attention in the form of Rochelle Gutiérrez' projects of rehumanising mathematics and mathematx.[19] Gutiérrez (2012) referred to problematisations of academic mathematics as White middle-class masculine knowledge. She argued that for 'most women, the working class, and people of color, a focus on dominant mathematics means that engaging in school mathematics largely require becoming someone else' and demanded a different kind of engagement with mathematics in which 'their participation will somehow change the nature of mathematics as a discipline' (p. 30). In a later publication, Gutiérrez (2018) introduced the idea of 'rehumanising mathematics'[20] in the sense that 'a student should be able to feel whole as a person—to draw upon all of their cultural and linguistic resources—while participating in school mathematics' (p. 1). In a different publication, Gutiérrez (2017a) focused less on education and more on mathematics as a scientific discipline. There, she pleaded for mathematx as 'a radical reimagination of mathematics, a version that embraces the body, emotions, and harmony' (p. 15). Gutiérrez countered Western essentialism with Indigenous epistemologies as the new basis of a practice that is pleasing, aesthetic, action-based, embodied, and diverse. Although she provided some examples of what that might entail, she leaves open the question 'which new forms of mathematics might arise' (p. 20). Elsewhere, Gutiérrez (2017b) commented more

19 IPA: [ˌmæθməˈtɛʃ], or *mathe-ma-tesh.*

20 To avoid misunderstanding, it should be noted that what Gutiérrez (2018) meant with 'rehumanising' does not directly respond to what I called 'dehumanisation'. While I presented dehumanisation as a denial of one's full humanity and a prerequisite of mathematics, Gutiérrez assumes 'that people throughout the world already do mathematics in everyday ways that are humane' (p. 2), but that this doing is denied by the hegemonial practices in the mathematics education classroom. I refrain from sharing this position, for Gutiérrez' list of such ways of doing mathematics (p. 4) reveals that we face what Dowling (1998) called 'celebrating non-European cultural practices only by describing them in European mathematical terms' or the recognition of 'a practice as mathematical only by virtue of recognition principles which derive from their own enculturation into European mathematics' (p. 14). Regardless of this point of critique, I find value in Gutiérrez' overall ideas.

carefully that 'we do not have good models for what a feminist, pro-Black/Indigenous/Latinx, socialist mathematics education would look like, or if even such a thing could exist' (p. 12).

I meet Gutiérrez' last comment with a good portion of pessimism. As Fischer (2006) and Bettina Heintz (2000) pointed out, a paramount social function and driving force of mathematics is the production of consensual knowledge and practice. The analyses of calculation and logic presented above concluded that both aspects of mathematics can be understood under the term of reaching consensus. The very nature of these techniques is that the individual is disregarded. Eventually, I would negate my earlier question if we can alter the *modus operandi* of mathematics so that dehumanisation leaves the equation.

In contrast to that, Gutiérrez (2017a) suggested that 'mathematx acknowledges that all persons will seek, acknowledge, and create patterns differently in order to solve problems and experience joy' and that 'multiple knowledges are valued and sought' (pp. 19–20). Apparently, mathematx would not be able to replace mathematics in its function of reaching consensus. A shift from mathematics to mathematx would mean that this practice loses its paramount, maybe even its entire function for society, thus making itself expendable. Could it be that mathematx would turn out as something completely different than mathematics? And if so, why then talk about *mathe*matx and not simply about an alternative epistemology? Or, asked differently, what of mathematics would be conserved in mathematx?

I propose that alternative epistemologies remain important for the study of dehumanisation through mathematics, because they help us to understand that our world can be understood differently. Such insights might not result in new epistemic forms of mathematics, but they might allow us to better capture the epistemological potential, limits, and dangers of mathematics. Admittedly, this perspective does not help us to counter epistemological discrimination in the mathematics classroom as was the initial attempt of Gutiérrez. We might come to find that we cannot wrench mathematics from the quills of White middle-class men that roam the history of the discipline. However, awareness of the particularities and political nature of the epistemology of mathematics, gladly aided by alternative visions of how to approach the world we live it, can help us to understand, support, or confront the ways in which mathematics contributes to dehumanisation in our societies.

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