Studies on Mathematics Education and Society

# BREAKING IMAGES

## ICONOCLASTIC ANALYSES OF MATHEMATICS AND ITS EDUCATION

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# 8. Human mathematics

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*This chapter discusses how we can think about mathematics as a human enterprise. It takes as its starting point the portrait of a European tradition that has considered mathematics as essentially a non-human realm. As a challenge to this tradition, a Wittgensteinian interpretation of mathematics as a special type of language among all the human languages is outlined and used to develop a platform for understanding mathematics as 'human mathematics'. This conception is finally given shape through two discussions, first through a challenge to the positioning of mathematics in our contemporary universities in close proximity to the natural and technological sciences. Instead, a narrowing of the gap between the sciences and the humanities with a consequent repositioning of mathematics in the epistemological landscape of our knowledge institutions is advocated. Secondly, a human mathematics conception is discussed in relation to learning and teaching. Connections are made to socio-cultural learning theory, and it is argued that the concepts of 'fog of mathematics' and 'centreless mathematics' can help in reconfiguring how to think about the learning of mathematics.*

## Introduction

Dominant stories told about mathematics are often linked to science and the certainty of scientific knowledge. Other more socially oriented stories about mathematics are related to our everyday practices in schools, homes, or the workplace. From a Wittgensteinian perspective – a perspective I shall discuss in the following – these socially oriented stories and practices, in conjunction, hold the truth about what mathematics is. This difference in perspectives and understandings of mathematics is the axis around which this chapter revolves.

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I will attempt to portray these two different perspectives in the following and discuss how there is an argument that the meaning of mathematics is nothing more than a social agreement in the use of signs we have developed over centuries – a purely human mathematics. This perspective highlights that non-human ontological ideas about mathematics could be utterly misleading and opens the discussion about socially oriented reinterpretations in the epistemology and ontology of mathematics. In this sense, the chapter deals with the question: What implications could a human and socially centred interpretation of mathematics have for our current practices?

My approach to getting closer to answering this question involves the following steps:

1. Give a short historical account of some of the central roles mathematics has played in our thinking about science and universities. This is a story dominated by the view that mathematics is non-human and represents the eternal structures of the world.
2. Present a language-centred philosophical position that argues mathematics can be understood as a multitude of human language constructions with many different types of uses and functions in our lives.
3. Discuss the opening of perspectives that presents mathematics as a completely social construction. I highlight two cases to illustrate this. The first case concerns university and mathematics – that is, how should mathematics be positioned in our epistemological and ontological landscape of sciences? In this case, the attempt is to give mathematics new interpretations in relation to the humanities as an expression of human creativity along the lines of poetry and literature. This is potentially a story of mathematics as exploring the limitations of (constructed) reasoning in the process of developing ever new and complex mathematical measures. The second case discusses how thinking about learning mathematics from the human mathematics perspective in general will differ somewhat from many traditional approaches. Thinking about the learning and teaching of mathematics under the

assumption that mathematics is a 100% social construction means that some principles can be highlighted to give direction for educational development.

With respect to point 2., I will draw upon the Wittgensteinian argument that mathematics consists of language games, which play many different roles in our lives, particularly in how we use, develop, and reach agreements on mathematical concepts. I will draw on Stuart Shanker's interpretation of Wittgenstein's social turn in the philosophy of mathematics, which carves out a specific position in the interpretation of Wittgenstein's writings on mathematics (Shanker, 1987). This is not a chapter that aims to persuade all critics of a thoroughly social interpretation of mathematics, but I will point to the main ideas and reasoning behind the position I call 'a human mathematics' in what follows.

## Order of the galaxy

Historical configurations of knowledge and mathematics have a huge impact on our understanding of the role played by mathematics today. Consequently, it seems reasonable to start thinking about the positioning of mathematics in our scientific worldview with an outline of some of the historical constructions that have surrounded mathematics. In order to trace some of the routes mathematics has traveled until today I will discuss aspects of its institutional connections to science and knowledge in a European context. The aim is to highlight dominant patterns of thinking about mathematics in the European history of ideas, being fully aware that there are many non-European historical stories that go even further back in time. The author of this chapter is of European origin and this unfortunately puts some limits on his insights into other historical trajectories. Accordingly, the following should be thought of as a local perspective about past ideas related to mathematics and how we could conceivably think differently about them in the future.

The local story inevitably connects to the highly influential interpretation by the Pythagoreans in Ancient Greece and the Academy built by Plato later on. The influence of this early interpretation of mathematics within a larger ontological framework can be traced in the medieval university structure, as presently discussed. And, in today's university, the idea of mathematics as an especially important element in exploring the world is, for example, reflected in the acronym STEM (Science, Technology, Engineering, and Mathematics) which at the same time disconnects mathematics from the sciences of language and social sciences.

By echoing this classical interpretation, one runs the risk of oversimplifying the actual historical complexities and entanglements and the connections between both modern and medieval scholarship environments and their Ancient Greek counterparts (see, for example, Høyrup, 1996, for an interesting account of these complexities). However, as Jens Høyrup suggests, there is no doubt about the dominant narrative concerning Ancient Greek mathematics and its impact on the European interpretation of the development of mathematics:

> This tale, more or less biased or false as an historical account, has none the less become material truth in the sense that it has contributed to the self-understanding and thereby to the cultural identity of the European mathematical community/communities for centuries. (Høyrup, 1996, p. 103)

However flawed this narrative is in representing the vast complexities of European and non-European origins of ideas about mathematics, its domination as a narrative is what matters for the argument of this chapter.

The Pythagorean interpretation of mathematics is often ridiculed as coming from a very speculative and religious environment. Nevertheless, the Pythagoreans are very importantly famous for connecting numbers and the relation between numbers to the heavenly spheres; in this way, they started a long tradition of relating mathematics to the structures of the universe and to the field of astronomy.

Archytas (428–347 BCE), at the time of Plato, explained how the Pythagoreans were the inspiration to connect the study of numbers to the study of the universe, as they 'handed down clear knowledge of the speed of stars and their rising and setting, and of geometry, arithmetic, and spherics and not least music, for these studies turned out to be sisters' (Archytas, cited in Pedersen, 1979, p. 20). Spherics was closely associated with what we would refer to as astronomy today and it was thought of as the materialisation of numbers in nature in its continuous form. Music was thought of as numbers in nature in their discrete form, while geometry and arithmetic were thought of as numbers in themselves—in both discrete and continuous forms (see Pedersen, 1979, p. 20).

The impact of the Pythagorean ideas about mathematics was established most forcefully by Plato. When he constructed his Academy, which later became an influential inspiration for the early medieval university, he found a central position for mathematics as a field that was of the utmost importance in the formation of thinking among his students. In many of Plato's writings, and especially in *The Republic* (Plato, 2022a), he outlines how the road towards a deeper insight into the many aspects of life can be furthered by prolonged studies of mathematics. In this way, Plato set the course towards putting mathematics on a pedestal among the sciences as the discipline that will train and strengthen reasoning and logical deduction. And it was notably a form of mathematics that was also considered as metaphysically connected to the order of things in the physical world.

When the first European universities were established in the eleventh century they were inspired by the Ancient Greek constellation and understanding of knowledge and their structuring of the different fields of study. In these universities, the faculties were normally the philosophical, the judicial, the medical, and the theological. To access one of the higher faculties one had to pass the bachelor exams in the philosophical faculty. Based on the tradition from the Pythagorean division of knowledge classification, these were ordered into 'seven liberal arts', divided between the study of Number (Quadrivium), with four subdisciplines, and the study of Letter (Trivium), which focused on grammar, logic, and rhetoric (Grane, 1991, p. 23).

Mathematics was connected by the Pythagoreans to the study of the universe, implying that the building blocks of the universe are of a mathematical nature. This conception underlines the idea that Letters are about human matters whereas Numbers are about the matters of the universe. Studies related to the Letter were, on the other hand, directly associated with handling human life and the social sphere. In this way, the deep gap in the scientific community today between STEM and not-STEM areas can be thought of as having been nurtured from this specific and speculative ontology in relation to the power of Numbers.

Plato himself was an active constructor in the reification of this constellation of knowledge. The key mathematical work to be handed down through history from Ancient Greece is Euclid's *Elements* and much of its content was inspired by scholars educated or situated in Plato's Academy. Archytas has already been mentioned and other famous examples are Theaetetus (417–369 BCE) and Eudoxus of Cnidus (395–342 BCE) who both researched at the Academy (O'Connor & Robertson, 1999).

To highlight the philosophical significance of the *Elements* for the times to come one can make several observations. First of all, within the specific Euclidean framework of mathematics – a celestial mathematics, you might say – the only permitted construction methods for constructing objects were the use of a ruler and a pair of compasses. In the Euclidean-Platonic epistemology of mathematics, these constructions represent eternal objects. Many students for centuries afterwards have been trained in these basic skills and the ruler and a pair of compasses were certainly to be found in the mathematics pupil's toolkit.

The first book of Euclid's *Elements* starts out by proving that one can construct an equilateral triangle from some basic actions of construction using the ruler and a pair of compasses. Many volumes later, the final proof in the final book of the *Elements* establishes the construction of the so-called Platonic Solids (Euclid, 1998, Book XIII). It is proven that there are exactly five of these solids, interpreted as representing the 'elements' (fire, earth, air, and water), with the fifth representing heaven and its twelve constellations. In other places, Plato relates the Platonic elements to the building blocks of all things and in this way makes a transparent connection between Euclid's *Elements* and his own Pythagorean and mathematically inspired ontology (Plato, 2022b).

In the last decades it has been more and more acknowledged that the early modern scientists like Isaac Newton and Johannes Kepler were much inspired by similar thoughts on mathematics. It is telling how Kepler describes how the regular polyhedra can be understood as the structure of the universe and here very much brings the ontology of the Ancient Greeks into the core construction of modern science. In his early work *Mysterium Cosmographicum*, he describes how the regular polyhedra in Euclid's *Elements* are to be conceptualised in an astronomical sense as the spherical structures surrounding earth (Kepler, 1596). In this way, he establishes a direct line to Plato's ideas about mathematics. He is echoed by the insights and ideas of Galileo Galilei on the study of nature as a realm where only mathematics can reach the deepest insights (Galilei, 1957, pp. 237–238).

After having stripped away the metaphysical connotations, universities today are very much aligned with the conclusions of this story. In the modern Humboldtian inspired universities, mathematics is often located next to physics and the natural sciences. This means that mathematics is still connected to the idea that it is the main tool for describing and understanding the physical world around us. The essence of this conception of knowledge amounts to something like the following: to understand human actions you must study letters and natural language and to understand the physical world, including the human body and its behaviours, you must use the numbers to get to the truth. And this is a relatively moderate interpretation of matters. The stronger interpretation goes along the lines that if you *really* want to understand any field of study you need the *hard* sciences defined by their use of quantification of the world through the Number.

## A social enterprise

In the previous section, I have tried to portray how mathematics has been interpreted as connected to the building blocks of the universe. It is a deep cultural heritage in Western inspired universities that mathematics is the language that can tell you the most about the world.

However, a contrasting perspective does exist, though it is much less dominant. In fact, numerous challenges have been raised against the idea of mathematics as a mirror of real-world structures, inherently tied to fields like physics, chemistry, engineering, and technology more than to other fields of knowledge. Among these challenges I will try to highlight a Wittgensteinian perspective that suggests that mathematics is a human construction through and through. From this perspective mathematics is not about mirroring the logical structure of the world but instead about creating a diverse mathematical language to use in a multitude of different types of social practices. From the many interpretations of mathematics as a social structure, I have consistently found Wittgenstein's interpretation, in his later works, to be both the most radical and the most credible. It is developed in the posthumously published *Remarks on the Foundations of Mathematics* but is also closely connected to his later principal work *Philosophical Investigations*. Wittgenstein's interpretation of anything is always subject to heavy debate and, as mentioned above, I will follow the interpretation presented by Shanker (1987) in *Wittgenstein and the Turning-Point in the Philosophy of Mathematics*. Explaining Wittgenstein's position in detail is beyond the scope of this chapter; the discussion is developed further in Ravn and Skovsmose (2019, 2020).

A social interpretation of language revolves around the idea that our words and sentences can only have meaning from a group of language users. In fact, Wittgenstein is famous for his argument that no single or isolated human would ever be able to pinpoint meaning within a word or symbol because there would be no group of users to discuss and reflect to what degree the use of the symbol or word would be correct. This is known as the private language argument in Wittgensteinian research, and it has been heavily discussed through the years (see, e.g., Candlish, 1998, and his discussion of Saul Kripke's notorious interpretation).

The argument is that only the use of symbols or words in a social group can establish the meaning of the symbol. In the Wittgenstein literature, this is known as the 'meaning is use' principle; to illustrate this, consider one of my favourite examples: the sign we make when we point our finger in a certain direction. This is a simple concept for adults to understand, and often across cultures. However, a young child might not grasp the symbolic meaning of the gesture in their early years, and may simply look just at your finger, regardless of the direction you are indicating. They do not know how to use this part of language and only gradually will they learn how to use this symbolic gesture.

The situation is similar in mathematics, according to Wittgenstein. When we are told to repeatedly add 2, we feel forced to write '2, 4, 6, 8, …' But what is it that compels us to do so? In Wittgenstein's interpretation, the only force at stake is the social training and large-scale practice in a community of mathematics users that, in the end, creates the sensation of the forced conclusion as being the most natural endeavour imaginable (Wittgenstein, 1967, p. 3e–6e). If we imagine the sequence '2, 4, 6, 8, …' written in chalk on a blackboard, Wittgenstein's point is that there is nothing hidden behind the chalk. The symbols themselves do not have any concealed meaning that could force us to act as we all do. Often this feeling of force has been attributed to logic, but according to Wittgenstein there is nothing supernatural occurring in logic or mathematics. When he rhetorically asks, 'In what sense is logic something sublime?' the answer is clearly 'in no way', and instead he presents the idea that mathematics is essentially collective agreements about rule-following in connection to specific symbols (Wittgenstein, 1997, p. 42e):

> Let us remember that in mathematics we are convinced of grammatical propositions; so the expression, the result, of our being convinced is that we accept a rule. I am trying to say something like this: even if the proved mathematical proposition seems to point to a reality outside itself, still it is only the expression of acceptance of a new measure (of reality). (Wittgenstein 1978, pp. 162–163)

But is this not a flawed position, as it might suggest that mathematics could then be arbitrarily agreed to mean anything? This arbitrariness is actually a cornerstone in Wittgenstein's interpretation. It highlights that his view of mathematics is as a language that, in its development, is not constrained or dictated by a sublime logic, nature, the universe, or anything else:

> But then doesn't it (mathematics) need a sanction for this? Can it extend the network arbitrarily? Well, I could say: mathematicians are always inventing new forms of description. Some stimulated by practical needs, others from aesthetic needs—and yet others in a variety of ways. And here imagine a landscape gardener designing paths for the layout of a garden; it may well be that he draws them on a drawing-board merely as ornamental strips without the slightest thought of someone's sometime walking on them. (Wittgenstein 1978, p. 99)

The mathematician is an inventor – a poet of the numbers one might say – one that slowly, in co-operation with a collective of other mathematicians, pushes the boundaries of what can be thought of as being rational in terms of measuring the world. Wittgenstein therefore agrees that mathematics is special, but not in the supernatural sense of revealing deeper or hidden dimensions of reality, unlike natural language.

> We feel that mathematics stands on a pedestal – this pedestal it has because of a particular role that its propositions play in our language games. What is proved by a mathematical proof is set up as an internal relation and withdrawn from doubt. (Wittgenstein 1978, p. 363)

What is special about mathematics is that it represents knowledge or grammar that has been established and removed from doubt. The task of the professional mathematical community is to push the limits for the ways in which we can measure reality and, in this process, to resolve any doubts about the rationality of these approaches. In this way the professional community of mathematicians has gradually constructed a grammar that supports science and calculations in everyday aspects of our lives.

This interpretation of mathematics is somewhat different from some of the other social interpretations of symbols of mathematics. In those views, mathematics is seen as resembling the empirical sciences by being based on extremely large amounts of empirical data and is, in principle, fallible, much like theories in physics or biology (as is well-known from the tradition following Lakatos – see, e.g., Hersh, 1998). Wittgenstein disagrees with this interpretation of mathematics. Mathematics does have a deep history based on human experiences, but this does not mean it has been forced upon us in any way by our surroundings. It would be more in line with Wittgenstein's ideas to say that mathematics is a tool in a large language toolbox that, for example, enables us to express empirical statements in the sciences. Mathematics is itself a measure or grammar, rather than the thing being measured.

This toolbox of mathematics has no clear boundaries – it can be about all sorts of measures relating to surfaces or statistics, strange types of numbers, and the many things that we cannot even imagine today that will come about in decades to come. The image of mathematics in this interpretation is one without a centre – mathematical concepts are continuously being developed and enriched by the structures and concepts surrounding them, and concepts of mathematics are constantly being renegotiated in minute details in everyday practices of mathematics users, but sometimes also on a major scale when new types of numbers, or the like, are introduced.

Wittgenstein was opposed to the interpretation provided by the influential logicians of the early twentieth century, including himself in the form of the so-called 'early' Wittgenstein. The outline presented above of mathematical development through gradual playing with the concepts is in stark contrast to his earlier thought patterns about logic and mathematics (Wittgenstein, 1983, first published 1922).

> It is interesting to compare the multiplicity of the tools in language and of the ways they are used, the multiplicity of kinds of word and sentence, with what logicians have said about the structure of language. (Including the author of the *Tractatus Logico-Philosophicus*.) (Wittgenstein, 1997, p. 12e)

The aim of these early twentieth-century logicians was to show that (scientific) rationality could have only one form and that this form was definitely not a human form but something humans had access to (in contrast to animals) through labour or through talent etc. By denouncing his own earlier work in the *Tractatus*, the later Wittgenstein presents a much more vivid and organically developing image of mathematics that is open for new paths and absolutely freed from an axiomatic limitation on how mathematics can develop as we know it from Euclid's *Elements*.

## If it's a language?

The above arguments and discussions are all of a historical and philosophical nature and one meets many of them in research communities again and again. Taking the position that mathematics is social through and through, will it really make a difference? That is the question that will be pursued now.

As described in the introduction I will try to imagine what difference could be associated with our mathematical practices. I am deeply inspired by the Wittgensteinian interpretation of mathematics; however, in the following I will go far beyond what Wittgenstein (or Shanker) could be held accountable for. I will delve into two aspects of what I shall call a 'human mathematics'. The first is the positioning of mathematics in the landscape of sciences in universities. The other aspect relates to the many learning situations that could be directly influenced by the social interpretation of mathematics.

## The position of mathematics in the university

Let us imagine that university faculties were up for reconstruction. Where would mathematics fit within the new landscape? We have seen how mathematics was historically tied to the description of the universe and nature and therefore placed close to physics and other natural and technological sciences.

Considering a human mathematics reconstruction of the university the positioning of mathematics could be quite different. The myriads of possible uses of mathematics today are related not only to the description of nature and the universe but, perhaps even more so, to human affairs and the structuring of the social sphere. This goes for the economy, infrastructure, working hours, tax systems, online presence, traffic, and so on. In many ways, the shift suggests that mathematics in practice also has a tremendous impact on almost all branches of the humanities and social sciences and that human life practices are flooded with numbers and measurements.

According to the human interpretation of mathematics, we should consider mathematics as being equally connected to the humanities, social sciences, and the natural sciences. Some might find this a disturbing or even threateningly invasive approach to repositioning mathematics, but it could also point towards something more fruitful. In the following we can consider different sub-elements of the discussion to qualify the issue.

First of all, there is a very famous cousin of mathematics often positioned in the humanities, namely logic (in accordance with the Trivium disciplines). Logic in many universities has been positioned within philosophy, where it is also closely related to the area referred to as theories of argumentation in philosophy. These studies within the humanities are not initially alienated from what we might call formal systems, including logic, mathematics, programming languages etc. From a Wittgensteinian perspective they could rightfully be called 'grammars' and in this way share family resemblances with the studies of the natural languages.

Second, some parts of the humanities and social sciences are very far from using formulae and mathematical expressions in their practices of research. This might be the case for literature studies, and some language and cultural studies, even though statistics or other applications of mathematics might be used in some approaches of these fields. However, while the scientific approach used in natural sciences focuses on numbers and quantifiable experiments, research in the humanities focuses on qualitative approaches. Humanistic research is never satisfied with counting or measuring but is in essence focused on establishing detailed narratives and rich interpretations about human culture under specific circumstances. This is known from approaches in phenomenology and hermeneutics as well as, for example, in organisational studies from post-structural perspectives. This means that Numbers can never be the focus or goal of all research. Mathematics in itself is a strong and diverse toolbox but it also has immense boundaries to what kind of knowledge and insights it can produce. Using only quantifiable measures in the world in research is only an extreme case of doing science that can reveal some things, but simultaneously it hides a lot of other things. In a possible narrowing of the gap between humanities, social sciences, and mathematics, mathematics must be given a clearer role in the scientific toolbox that holds a myriad of qualitative as well as quantitative approaches and attempts to merge or overlap approaches from these two main categories. To do better research overall, a landscape of scientific approaches much broader than mathematical tools is highly needed for both traditional studies in the STEM area as well as in the humanities and social sciences.

Third, it is interesting to discuss mathematics as an outdated science. I am hinting at the perspective that with the invention of computers – and the use of them in, so to speak, all practices in modern societies – new sciences have been constructed that are closely related to formal languages in new ways. Computer science is the broad term for the many logical studies that bring formal languages closer to practical use, whether it is used in a hardware or software product, or even in the theoretical underpinnings beneath the World Wide Web and other platforms of social interaction. It is from this perspective quite easy to get the idea that mathematics is more than anything else a cultural and historical phenomenon.

Fourth, there are ways in which mathematics portrays family resemblances less with formal approaches but more with the creation of language as we know it from literature and poetry. In the human-oriented interpretation of mathematics, the research component has the task first and foremost of developing new measures to be used in different aspects of life and of demonstrating their practical sensibility. In a way this resembles treading new paths in literature and poetry. Literature and poetry are developed both within well-known schematics or new forms of media that are used to help us think about reality by challenging everyday conceptions or playing with new meanings of concepts. In many ways, this could be thought of in parallel to the developments in mathematics. In this way, mathematics could be understood as a language in constant development, in parallel to literature where we essentially search for new ways of interpreting the world around us.

Finally, the division between the humanities and social sciences, on the one hand, and the natural sciences together with the technological disciplines, on the other, is a well-known issue of two scientific cultures. Often reference is made to the work of C. P. Snow (1993) who discussed how these cultures are in opposition to each other. The argument I will make is that the interpretation and positioning of mathematics is at the centre of this cultural dispute. As long as mathematics is seen as a secure foundation beneath all 'real' (read STEM) knowledge, then the humanities and social sciences will remain in the periphery of what is recognised as truly scientific science. The argument that mathematics is a purely human language formed in connection to the world around us, implies that the humanities are, in fact, a natural destination for thinking about mathematics. Languages have histories and are produced under the pressure of political and social circumstances that need to be understood in order to understand languages and their use – even in the case of formal languages.

This imaginative discussion of the positioning of mathematics leads me to the conclusion that narrowing the gap between the two scientific cultures necessitates a deeper reflection about the nature of science itself. This reflection should incorporate both mathematics as we know it today and qualitatively oriented research approaches in a collective narrative about the diversity of science and scientific research approaches. Wittgenstein describes how mathematics is placed on a pedestal for a specific reason. However, we need a much broader area of expertise than formal languages to be put on that pedestal in order to establish the most insightful knowledge creation and dissemination in universities and beyond.

## Learning mathematics from a social perspective

In the landscape of learning theories, a dominant position is the so-called socio-cultural learning theory championed perhaps most forcefully by Etienne Wenger (1999) with the concept of 'communities of practice'. In many ways this theory of learning can be understood as an extension of the social aspects of learning that a human mathematics could propose. According to this theory, key dimensions in a learning process relate to identity building from participation in a community of learners. Each individual needs to travel the distance from the periphery of the social practice to its centre in order to become more and more proficient in the specific practices of the community. Learning in this framework has an extremely high focus on the community of learners as opposed to the individual. To learn something is to become a member of a certain social group and know their ways and behaviours. This is in stark contrast to an idea of teaching and learning that is focused on delivering clear logical packages of knowledge to a sole and rational learner. Instead, the idea is that all knowledge is so incorporated into social practices that learning content itself cannot be disconnected from being an active member of the practice. Learning mathematics is identity construction and is about becoming enculturated into the practices of mathematics.

In everyday school practices the social understanding of learning and mathematics will inevitably entail a strong emphasis on participating actively in mathematical practices. It is crucial to speak the language oneself in co-operation with fellow students and guided by teachers who are individuals that carry the social practices of mathematics as part of their identity. This way of thinking about mathematics learning and teaching is therefore alienated from an approach that tells students the 'result', so to speak, on a blackboard, based on the hope that a logical ability located in the skull of the individual will give them an 'aha' experience about the right way to prove or calculate something. The direct route to learning a particular part of mathematics is involvement in the actual practice of transforming chalk while discussing and evaluating with peers and strong community members.

One principle that should be highlighted is what I will term the *fog of mathematics* (drawing on the idea of the 'fog of war' in many computer games, where only parts of the map are visible at any given moment – the rest of the map is hidden in fog, and you can only guess who is where). Under the social interpretation of human mathematics outlined above, students have no access to a logical faculty of the brain or something of the sort. We have touched upon the example of pointing a finger in a certain direction in parallel to the situation where you need to 'add 2'. For the student unfamiliar with adding 2 ad infinitum this could mean many things. At first, the teacher may experience that things are going as planned – 2, 4, 6, ... – but then when the student reaches 20, she starts to add 2 twice – 18, 20, 24, 28, ... etc. There is absolutely nothing except the community of practitioners that can tell how the fog of war should be cleared. When one first enters a new practice, only imagination and familiarity with similar practices can advance understanding beyond solitary efforts. The only and final test of truly grasping a concept or practice lies in how it stands up to scrutiny and feedback from the experienced language users in the mathematics community.

Wittgenstein's argument on this topic is that no sign itself holds information about its own meaning. Even the simplest of signs in mathematics like '1', '2', '3', ... are completely open for interpretation in so far as a community has not clearly stated how to proceed and, even then, there will be millions of possibilities for misinterpretation of the proposed decided meaning of a concept or symbol.

In this way the human interpretation of mathematics deletes any notion of the contemplative approach to learning mathematics or studying mathematics. Mathematics is not located in the individual. Instead, being capable of doing mathematics means to be able to participate in communities of uses of different kinds.

This participation in a mathematical community points towards what can be called the principle of a *centreless mathematics* (in the sense of there being many equally important and complex mathematical practices). One community of mathematics is located in first grade, and another is found in a discussion on vector spaces at an international conference. These communities have family resemblances in the way they possibly share some symbol transformation, argue by writing on the blackboard, or present mathematical themes to their peers. But they are all also obviously incredibly different. Wittgenstein tries to highlight different practices that we relate under the same area or concept as for example 'mathematics' as having family resemblances (Wittgenstein, 1997, p. 32e). In this way doing mathematics is something that is deeply dependent on the context in which it is conducted.

Even in the much more closely related communities of mathematical practice, such as first grade mathematics and seventh grade mathematics, the concepts and the use of symbols do not have the same meaning. In first grade, '2' is the number focused on. In the seventh grade, the meaning of '2' has been developed into '+2' because it has been incorporated into a context including negative numbers. The meaning of mathematical concepts in this sense differs across the many practices where they are used even in the fairly similar contexts of school classes. And remember that in the interpretation of human mathematics there is no 'real' version or story about the number '2'. The meaning of '2' in the first-grade classroom is just as valid and just as valuable as the meaning of '2' in the international conference room. There is epistemologically and ontologically no true '2' to gravitate towards. You might see the conference '2' as more complex or further developed or more precise but there is no non-human reality to measure against, and this puts mathematical practices on an even footing, ontologically speaking.

## Summary

In this chapter I have discussed several aspects about mathematics under a human mathematics interpretation. I have explored reconfiguring mathematics in relation to its position in the broader landscape of sciences and its influence on how we think about learning mathematics. The discussions are obviously only initial steps towards reshaping our notion of mathematics. They are also connected to many other discussions, for example to the problems in academia and beyond of putting quantitative research on a pedestal.

Another connected discussion relates to what, in a Danish context (the author's main frame of reference), is referred to as the distinction between the hard and the soft sciences. This normative description of sciences is used without any hesitation far too often. This chapter is also an attempt to reflect on what constitutes 'soft' versus 'hard'. According to my interpretation of human mathematics, disciplines like mathematics, physics, and technology may be considered a lot ‘softer’ than typically assumed, especially in the context of funding allocations between STEM fields and the humanities and social sciences. In fact, ‘soft’ might actually encompass some of the most challenging aspects of both life and research, following a human mathematics interpretation. We need a more balanced scientific landscape that will make dichotomies like these irrelevant and here our understanding of mathematics plays a key role.

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