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BREAKING IMAGES

ICONOCLASTIC ANALYSES OF MATHEMATICS AND ITS EDUCATION

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17. Bringing ethnomathematical perspectives into classrooms

Swapna Mukhopadhyay and Brian Greer

In this chapter, we offer some suggestions, informed by our personal histories and experiences, as to how ethnomathematical perspectives might enrich school mathematics classrooms. We regard this as inherently political work, in terms of combatting the intellectual White supremacy that pervades the Eurocentric narrative of the history of academic mathematics and that is explicitly or subliminally everpresent in so many mathematics classrooms. Likewise, we argue that the ongoing worldwide homogenisation of school mathematics is unhealthy. Above all, we argue that school mathematics is culpably deficient in terms of its relations with other forms of mathematical activities and insofar as it does characterise such relationships, often harmfully misleading.

Introduction

> Ethnomathematics is the mathematics practiced by cultural groups, such as urban and rural communities, groups of workers, professional classes, children in a given age group, indigenous societies, and so many other groups that are identified by the objectives and traditions common to these groups.
> (D'Ambrosio, 2002, p. 1)

As Ole Skovsmose (2022) establishes, the concerns of Ethnomathematics are intimately related to those of critical mathematics education in general.

We begin by drawing out of our backgrounds the insights that inform our current perspectives. As presently described, Swapna's engagement goes back to the 1980s, her doctoral research being conducted in a village in India, studying children learning in a way very different from being

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schooled, and at a time when it was becoming clear to anthropologists and cultural psychologists that ascribing universality to local European descriptions and theories of humanity is absurd. During the 1980s and 1990s Brian was moving from studying mathematical cognition, to mathematics education, and thence to critical mathematics education. He heard Ubiratan D'Ambrosio speak for the first time in 1995 and soon after that the authors of this chapter began to work together, as sketched below.

We summarise our engagements with Ethnomathematics across our careers and relate these to what we see as important themes. Against this background, we pose the central question: In what ways, given the realpolitik of educational regimes across the world, could school mathematics classrooms be enriched by ethnomathematical perspectives appropriate to the particular socio-political contexts? We mainly address classrooms in the United States (but of course, with relevance, *mutatis mutandis*, to many other educational systems). Accordingly, this chapter contrasts with Aldo Parra's (Chapter 10, this volume) which is about directly collaborative political work with communities in South America; however, in the current state of educational regimes almost anywhere, any attempt to infuse school classrooms or initial preparation of teachers with an ethnomathematical perspective is a political act. Such work presents both challenges and opportunities (Greer, 2021; Vithal & Skovsmose, 1997).

The experiences that we relate and reflect upon are illuminating for several broad themes that we highlight. A central thrust in the inherently political nature of Ethnomathematics has always been the construction of a counternarrative to the Eurocentric myth about the development of academic mathematics, a myth that can be seen as a long-established, deeply-entrenched, and ongoing manifestation of White (intellectual) supremacy. We also address the air of intellectual superiority of the mathematical academy towards what we have termed 'the mathematics of people who make things that work' (Mukhopadhyay & Greer, in press).

Much more attention needs to be given to establishing links between school mathematics and the day-to-day lived experiences of students, their families, and communities. And the paraphernalia of the technological age, with all their associated opportunies and dangers, form a pervasive part of their lives.

Ethnomathematical studies

Studying the children of weavers, potters, and farmers in rural India

During the 1980s, when Swapna was working on her doctorate at Syracuse University, it was becoming increasingly clear, through the work of scholars such as Jerome Bruner, Michael Cole, and many precursors, that cognition is never culture-free and that, for example, Piagetian tasks should not be viewed as universal windows into children's cognition.

The fieldwork for Swapna's doctoral dissertation was carried out while living in a small village in India near the Bangladesh border. As part of the preparation for this work, she took classes in weaving and pottery, the latter being something she still does. In the village, she totally embedded herself in the lives of the community, who looked after her with great care. Coming from a middle-class big-city background, the experience was formative in so many ways, not least in leading her to respect the knowledge, skills, and adaptability of the people and to see a very different way of growing up, in which the children, from the earliest years, learn by involvement in the family's work – what Andrew Dayton and Barbara Rogoff (2016) characterise as 'learning by observation, participation, and invention'.

Specifically, she wanted to explore how this early experience might be reflected in children's cognitive, in particular spatio-mathematical, functioning. The children were aged eight to twelve, and the expectation that their contributions to the family would take precedence over going to school, meant that their exposure to schooling was minimal. The form of weaving, carried out by looking down on a two-dimensional surface contrasts with the three-dimensional pottery, made on a wheel. It was confirmed that the weavers' children performed relatively better on two-dimensional tasks and the potters' children on three-dimensional, with both groups outperforming farmer's children on spatio-mathematical tasks in general. All groups performed computational tasks embedded in story problems at comparable levels.

As one example, when she posed conservation of liquid volume tasks to girls, they accurately predicted the level that the liquid would rise to in the second container, reflecting their experience and responsibility in buying cooking oil poured into a container at the market and ensuring the price was fair.

Thus, in many respects, the work illuminated the great differences between a schooled childhood (in which learning is separated from life-related consequences) and learning while doing, though graduated apprenticeship, and with consequences for actions.

The boat-builders of Frasergunj

During a trip to the Bay of Bengal some fifteen years ago, we visited a small village called Frasergunj, where we came across a group of men building a large wooden fishing-boat. Throughout the period since, Swapna has formed a close and ongoing friendship with these craftsmen, while conducting extensive observations and interviews to try to understand how they make such complex and well-constructed boats (Mukhopadhyay, 2013).

A team of eight to ten men, of varying age and experience build a boat in for to six months during the dry season. They are carpenters, mostly unschooled and illiterate, from villages in Bangladesh, who have adapted to the specialist skills of boat-building. They work almost always without blueprints – when asked about this, they told Swapna that they can work to plans if required but 'it slows us down'. While working, there is very little speaking. Less experienced members of the team are mentored, often without speaking, and learn by doing.

When asked about how they know that some part of the process has been done correctly, for example, curving a plank by heating and pulling over a fulcrum so that it can be added in the progressive construction of the hull, their responses hardly go beyond saying something like 'we can see it'. And when asked how they judge if a boat is a good boat, they answer along the lines that it keeps the fishermen safe and lasts a long time – in other words, it works.

We do not suggest that boat-building as a topic could naturally be introduced into mathematics classrooms unless the context is appropriate – for example, Jerry Lipka and his team (Lipka, Wong, Andrew-Ihrke, & Yanez, 2012) have included canoe construction in curricular materials they developed and used with children of the Yup'ik people of Alaska. The implications are at a more fundamental level. Is there a place for teaching mathematics less in the typical style wherein the motivation for learning something is that it will be useful for learning more mathematics (and that justification can be repeated indefinitely) or for some vague work-related purpose? At the most general level, as discussed further below, could school mathematics be more relatable to the students' lives? At a theoretical level, what are the implications of the distinction made by Edwin Hutchins (2000) between 'cognition in captivity' and 'cognition in the wild'.

Tlingit culture: 'Sharing our knowledge'

A more recent formative experience for Swapna has been working with the Tlingit people of the Pacific Northwest coast of the United States, mainly in South-West Alaska. The Tlingit share the history of oppression of all Indigenous Americans, North and South, through cultural genocide, linguicide, exploitation, and intergenerational trauma. As well as being subjected to oppression by Europeans, they suffered at the hands of the Russians, from whom the United States 'purchased' Alaska in 1867.

With a population of about 600000, and spread across the border into Canada, present-day Tlingits, with the neighboring Haida and Tsimshian, strive to reclaim their cultural identity and pride. Swapna has worked closely with the Sealaska Heritage Institute in Juneau.[1] In particular, she has collaborated with Tlingit weavers and basket-makers in Juneau, Sitka, Hoonah, and with school educators, to explore how the ethnomathematical perspective might enrich the school experience of Tlingit children. For example, she has interacted intensively with the legendary Haida master-weaver, Dolores Churchill, with whom she co-taught professional development classes for teachers. Of interest is that, while Swapna's attention is drawn to the complex geometry of the finished designs, the expert weaver focusses on the line-by-line

1 See https://www.sealaskaheritage.org

creation, with a primary emphasis on counting. Numerous aspects, such as the conception of the finished product the creator has during the act of making it, and how expert knowledge is passed on, remain to be understood, and underline how ethnomathematicians must exercise caution when projecting formalised mathematics on to what are not *primarily* mathematical activities.

Another fascinating aspect is the deep ecological consciousness embedded in the complex Ethnoscience involved in the preparation of materials for basket-making (Mukhopadhyay, 2009). Prior to European contacts, these finely woven watertight spruce root baskets were widely used for food preparation by submerging heated rocks with edibles to be cooked.

Swapna was indeed very fortunate to teach summer sessions for local teachers with eminent Tlingit and Haida local scholars and elders. One of these experiences focused on STEAM (Science, Technology, Engineering, Art, Mathematics) curricula examined the evolved design and construction of traditional halibut fish-hooks (which work).

Tlingit artefacts, whether practical or ceremonial, are always for use and complexly decorated. Bilateral symmetry, whether in canoe construction, realistic carvings of living creatures for totem polls, abstract patterns in weaving and basketry, is pervasive. Moreover the concept of symmetry, related to balance, has symbolic importance within the culture, though the Tlingit language does not have an equivalent word. The Tlingits are divided into two moieties, the Eagles and the Ravens, and whenever any discussion is taking place, there is an agentive expectation that if an Eagle or a Raven speaks, he or she must be balanced by a spokesperson for the other moiety.

We have been fortunate to attend a number of Tlingit Clan Conferences in Juneau, which combine cultural events with academic papers on important issues such as the effects of intergenerational trauma, and with a major emphasis on language revival, particularly among young people; also related gatherings in various places under the wonderfully appropriate title ‘Sharing our Knowledge’, reflecting a relational conception of the interactions between cultural knowledge systems (Parra, this volume).

Digging where we stand

Teacher preparation

Throughout her career as faculty at various universities working with future teachers, Swapna has introduced students and others to the ideas of Ethnomathematics through activities in and beyond class. In particular, she developed a strong relationship with Portland Art Museum, which displays rich examples of Native American culture artefacts, and which has an outstanding outreach to public education. By taking students to the museum on field trips (once holding classes for an entire term in a room within the museum), she focused on students learning to see and analyse the mathematics in typically unconsidered contexts. Students were encouraged to bring their children and other family members to such field trips. In particular, the native artefacts, both decorative and functional, are steeped in bilateral, translational, and rotationsl symmetry (Washburn & Crowe, 1987). By asking students to examine those symmetries, and to create new designs of their own (for example by making printing blocks for generating symmetry-rich designs) she introduced formal analysis of symmetry, a topic that forms part of the national mathematical framework.

Portland Art Museum, like many museums, is going through the process of re-examining their roles. One aspect in which they have shown leadership is through exhibitions that demonstrate how contemporary Native American painters, weavers, photographers, fashion designers, and so on are establishing relational bridges with other modern traditions, thereby negating the image of Indigenous peoples as belonging only to the past.

Culturally Responsive Mathematics Education

In 2004, under the auspices of the Centre for Learning and Teaching—West, a program funded by the National Science Foundation (NSF) spanning Portland State and four other universities, a conference was held at NSF headquarters with the above title. Participants included many of the most prominent figures in mathematics education taking, broadly speaking, an ethnomathematical stance.

We took responsibility for editing a book reflecting contributions to that conference and it duly appeared, with Arthur Powell and Sharon Nelson-Barber also co-editing (Greer, Mukhopadhyay, Powell, & Nelson-Barber, 2009). As far as we are aware, the specific term 'culturally responsive mathematics education' originated in this endeavour. We illustrate the nature of the book by reference to two of the contributions.

Geneva Gay's (2009) contribution begins with analysis of reasons why many teachers have an image of mathematics as culture-independent thereby absolving themselves of the responsibility of knowing how to teach in a culturally responsive manner. Rejecting that position, she asks a question of central importance in the United States context, marked as it is by large populational discrepancies between students and teachers: 'How can middle-class, monolingual European American math teachers work better with students who are predominantly of color, attend school in poor urban communities, and are often multilingual?' (p. 189).

Lipka worked with the Yup'ik people of Alaska for four decades (Lipka, Yanez, Andrew-Ihrke, & Adam, 2009; Lipka et al., 2012). Much of his work focused on curricular development, the creation of modules linking standard mathematical content to culturally-situated activities such as canoe construction, house construction, star navigation. The account he gives (e.g., Lipka et al., 2009) makes clear how his research program was carried out very much within the rules of the system. The bottom line was that in order to secure and retain funding for research it was necessary to produce statistically significant results from rigidly designed experiments providing evidence that the approach used yielded better outcomes *as defined by test results*.

However, in retrospect, Lipka (2020), in reviewing the intersection between Indigenous knowledge from various cultures and attempts to escape catastrophe through climate change, concluded that he needed to make a significant paradigm shift from 'ethnomathematics [in] the school context to the larger social-cultural-ecological systems' thus bringing him close to the political involvement advocated by Parra (Chapter 10, this volume) and exemplifying environmental issues as one arena where promising collaborations between formal science and Indigenous knowledge are happening.

Alternative ways of knowing (in) mathematics

At Portland State University, starting in 2006, Swapna organised a public lecture series, attended by appropriately diverse audiences, under the title 'Alternative forms of knowledge construction in mathematics'. Most of the invited speakers contributed chapters to a subsequent edited volume:

> This book is about the celebration of diversity in all its human form, specifically in relation to mathematics and mathematics education: culture, ethnicity, gender, forms of life, worldviews, cognition, language, value systems, perceptions of what mathematics education is *for*. (Mukhopadhyay & Roth, 2012, p. 1, italics in original)

In this book, the contribution by Mariana Kawall Leal Ferreira (2012) exemplifies her intense political involvement with Indigenous peoples in Brazil (see especially Ferreira, 2015). The chapter contributed by Gary Urton (2012) exemplifies the interplay between mathematics and politics in the context of colonisation, in this case the Spanish invasion of the Inkan Empire. Specifically, he describes how the highly developed statecraft of the Inkas, using khipus to record data, was forcibly replaced by double-entry bookkeeping in the European style. This exemplies a theme all too common in the history of colonialisation, the suppression of often superior cultural practices by the invaders (several examples, such as navigation, are described by Raju (2007). It is very important that, following colonisers' suppression of alternative knowledge in, for example, environmental science, botany, medicine, we are beginning to see extremely important collaborations across such knowledge systems. Without elaborating, we suggest that in the case of mathematics, the historical trajectory has been different, taking more strongly the form of assimilation.

Culturally Responsive Elementary Mathematics Education (CREME)

In 2014, the Oregon Department of Education funded a dozen or so projects under the Culturally Responsive Pedagogy and Practices Grant. Our project was the only one dealing specifically with mathematics, and it focused on elementary school students because of the importance we attach to the foundational impact of the early years. It was based on intensive interactions with a small number of dedicated teachers and their students in two Portland schools. We were fortunate to have as supportive advisers Danny Martin, Geneva Gay, and Marta Civil.

We worked mostly in two contrasting schools in North Portland. Rosa Parks Elementary, and Trillium Charter School. The students at Rosa Parks (though not the teachers) are predominantly African Americans, migrants, and refugees from many countries in Africa and elsewhere. Throughout our time working there, the school was under severe pressure on account of test scores being low. Trillium, an independent charter school (rather than the profit-oriented or religiously based kind), had relative pedagogical freedom in the elementary grades since the performance of students in the later grades was good. Thus, teachers enjoyed a considerable amount of autonomy, as illustrated below.

The project proved extremely educational and fulfilling for us. Ten teachers engaged fully, and we owe them a great deal. In the process, we learned very clearly about the power of the systemic straightjacket limiting teachers at Rosa Parks, an aspect well documented by Alan Schoenfeld (see Chapter 14, this volume)—above all, how testing imposes constraints.

The overall organisers held regular meetings at which representatives of all the projects reported on their work and we exchanged views. We became aware of a degree of tension between our aims and those of the organisers as interpreters of the funders' requirements; let's just say we were not in sympathy with the crude requirement to demonstrate rises in test scores as evidence that our approach had merit.

We also learned about the daily reality of the students. One of the teachers was asked why a student was sleeping and explained that he could not sleep at night (in a car) because the street lights were too bright. We tried hard to communicate with families, following the inspiring work of Marta Civil (e.g., Civil & Quinteros, 2012) and to channel Luis Moll's concept of 'funds of knowledge' which is 'based on a simple premise: People are competent, they have knowledge, and their life experiences have given them that knowledge' (Gonzalez, Moll, & Amanti, 2005, p. ix). In these attempts we encountered substantial obstacles, including the natural guardedness of immigrants and refugees in the current political climate.

Nevertheless, on looking back, there were many highlights in our rich reactions with students and teachers. A chapter co-authored with the teachers begins with a vignette about the power of simply asking students to find out and report to their class how to count up to twenty in languages spoken at home (Ford et al., 2018, pp. 169–170). One of the teachers (Koopman, 2017) conducted a project which began by students checking the labels on their t-shirts to see where they were made. These data were recorded on a world map. Koopman then told the students how the t-shirt was introduced as a working garment at the beginning of the twentieth century. He provided them with a mass of data relating to the economic lives of the workers and their families (most of whom also worked) in relation to the cost, at the time, of essential items. With these data, the students analysed family budgets. He also elaborated on the union movement, and concluded with some data on contemporary sweatshops in Asia and elsewhere.

A major element of CREME, linked to mathematics where possible, was to counter deficit models with support for student identity and agency. For examples, students recorded lists of what they could do ('I can bath my baby brother, I can do multiplication, I can skateboard backwards') on strips of paper rolled up and kept in containers they could hang around their necks, which we called 'talismans'. In the same vein, students (with minimal guidance from teachers) drew self-portraits and wrote poems beginning 'I am from' about themselves and their families (often in languages other than English). These were collected and self-published in three volumes 'Where we are from', 'We are from', and 'We are' with book-launches attended by parents and communities of the two schools at which children read their poems.

Themes

We next identify major themes that we can illustrate from our experiences as described in the previous section. Overarching all is the intimate relationship between forms of capitalism and the political enterprise that is (mathematics) education. Gay (2009, p. 194) expressed it forthrightly: 'mathematics becomes a proxy for academic racism, ethnic inequities in educational opportunities and a means of perpetuating a class system of "haves" and "have nots"'. Although the reader should catch many resonances in what follows and elsewhere in this book, that is too massive a topic to address adequately in this chapter.

Bringing more evidential rigour to the history of mathematics

Within Ethnomathematics, since the outset, a prominent theme has been the construction of a counternarrative to Eurocentric claims about the development of academic mathematics, more specifically what Jens Høyrup (1992) calls the 'Greek myth'. Many of the contributions in the seminal early compilation by Arthur Powell and Marilyn Frankenstein (1973) address this challenge.

What might be called the social construction of 'Pythagoras' is an appropriate starting point. According to a leading student of Greek mathematics 'Pythagoras the mathematician finally perished AD 1962' (Netz, 2003, p. 272). The date refers to a book by Walter Burkert (1962/1972), which is heavily cited in the entry on Pythagoras in the online *Stanford Encyclopedia of Philosophy* (Huffman, 2018). In surveying historical writings relating to Pythagoras, it is notable that forgeries are mentioned thirteen times. The article ends by stating that the consensus among scholars is that *Pythagoras was neither a mathematician nor a scientist.*

At the systemic level, Morris Kline provided a definitive statement of the Graecocentric myth:

> Mathematics is a living plant [that] finally secured a firm grip on life in the highly congenial soil of Greece and waxed strong for a brief period. In this period it produced one perfect flower, Euclidean geometry. The buds of other flowers opened slightly [...] but these flowers withered with the decline of Greek civilization, and the plant remained dormant for one thousand years. Such was the state of mathematics when the plant was transported to Europe proper and once more embedded in fertile soil. (Kline, 1953, p. 27)

Important counters to this absurdly extreme position have been provided by, for example, George Gheverghese Joseph (1991), Chandra Kant Raju (2007), Jim Al-Khalini (2010), and throughout the work of serious historians of mathematics such as Høyrup (see Greer, 2021, for an overview of his work and its implications for mathematics education). By way of example, there is strong evidence that important elements of calculus were developed in India before Gottfried Wilhelm Leibniz and Isaac Newton (Joseph, 1991), that the channels of communication existed to carry that knowledge to Europe, and that Eurocentric historians of mathematics have sought to suppress consideration of such evidence (Raju, 2007).

Contemporary educational politics in the United States

Without taking the space to elaborate, we assert that the position so clearly enunciated by Kline in the quotation above can be seen as an intellectual component of the White supremacy that is evident in the contemporary United States and many other parts of the world. The provocations of ethnomathematicians and other critical mathematics educators in opposing this position have contributed to the dragging of mathematics education into the 'culture wars' within the United States. Relative to the earlier 'math wars' described by Schoenfeld (Chapter 14, this volume), these developments are taking an extreme form, including personal attacks on individuals (Boaler, 2022).

A pervasive problem is illustrated by Jo Boaler with a quotation from a Nobel-prize-winning physicist:

> When I talk about education, I frequently have physicists lecture me on how I am wrong [reflecting that] nearly everyone [...] believes that they are an expert on education, just by virtue of having been to school or having a child who has attended school. (Wieman, personal e-mail message, cited by Boaler, 2022)

This rings true, above all, in relation to mathematics education. We offer the observation that a great many people, in particular politicians and educational administrators, will pontificate on the importance of children being able to, for example, solve quadratic equations, while themselves being unable to do so.

The mathematics of people who make things that work

The above phrase refers to another massive swath of humanity whose expertise is generally looked down upon by mathematicians and, at least until recently, largely ignored by historians of mathematics. In ancient Athens, it has been pointed out that:

> A consensus has emerged that Greek mathematics was heterogeneous and that the famous mathematicians are only the tip of an iceberg that must have consisted of several coexisting and partially overlapping fields of mathematical practices. (Asper, 2009, p. 107)

Ray McDermott (2012) alludes to many examples showing the falsity of the 'official story' of the role of mathematics in work. A particularly striking example is a report on ethnographic studies of architects:

> In nearly a year of a fieldwork at the architecture firm, I never saw any of the architects write down an equation and then manipulate it. I rarely saw calculations more complex than the basic arithmetic operations. The extent of geometric practice was a deep familiarity with shapes, often Euclidean ones, and an ability to visually and physically transform them in design practice. (Stevens, 2010, p. 83)

This does not sound so different from the boat-builders described above!

Two other working architects are described in vividly contrastive terms:

> 'Stupid Gerry' was terrible at the fractions examination he had to pass to become a professional draftsman, BUT he was smart and excellent at drafting, even at the parts that seemed to require a green-thumb knowledge of fractions [...] 'Dumb Ted' could be found looking incompetent wherever teachers or other students were pushing mathematics [...] but his work was excellent as long as he did not have to solve mathematics problems on a test. (McDermott 2012, p. 86)

Diversity versus homogeneity

Humanity is diverse in multiple ways, including language, ways of life, epistemologies, knowledge systems, ways of conceptualising and interacting with the environment. Arguably, awareness of this multidiversity is being eroded by globalisation (Westernisation), as warned against earlier in a study of the spatial epistemology of the Navajo people:

> Through a systematic superimposition of the world view and thought system of the West on traditional non-Western systems of thought and action all over the world, a tremendous uniformization is taking hold [...] The risks we take on a worldwide scale, and the impoverishment we witness is – evolutionarily speaking – quite frightening. (Pinxten, Van Dooren, & Harvey, 1983, pp. 174–175)

Ethnomathematics points to mathematics being embedded in multiple families of activity systems. In simplistic terms, we can point to those situated within: formal (academic) mathematics; mathematically-suffused cultural practices; mathematics of work; 'everyday mathematics'; school mathematics (see Harouni, 2015). With the exception of the first and last (to a degree), each of these is a very fuzzy category.

The diversity within formal mathematics, illustrating that it is open-ended, and that its history extends to the present and future, is illustrated, for example, in Hersh (2006). That there is enormous diversity within the next three families is obvious. By contrast, school mathematics, locally and globally, is increasingly characterised by homogeneity – converging towards a monoculture, to use the term of Mark Wolfmeyer (Chapter 16, his volume), with curricular frameworks and global testing (Chapter 15, this volume) as major instruments. It is also characterised by relational isolation from other families of mathematical activities. Indeed, we argue that, as exemplified in the discussions above about the history of mathematics, about mathematics in work contexts, and about mathematical modelling of human situations, insofar as those relations are promoted, they are often misguided and harmful.

Another aspect of diversity relates to terminological usage that may have consequences in framing images. It is understandable that a great deal of writing to date refers to 'Asians', 'Europeans' and so on, but surely it is time to move beyond such essentialising. Swapna, as an Indian, feels little in common with the Japanese, for example, while Brian is acutely aware that the Irish, as well as contributing to White supremacy, were also the victims of English oppression. In a similar vein, we regard it as dangerously superficial to refer to, say, 'African culture' given the diversity of African cultures. Pre-European peoples of North America, such as the Tlingits and the Navajo, are as diverse as the environments to which they are adapted. And it is surely possible to improve on the nonsensical term 'Western mathematics'!

Lived experiences of children, their families, and communities

Django Paris and H. Samy Alim (2017, p. 7) point out that 'contemporary linguistic, pedagogical, and cultural research has pushed against the tendency of researchers and practitioners to assume static relationships between race, ethnicity, language, and cultural ways of being'. This protest is echoed by Indigenous peoples who reject the all-too-common characterisation of them in terms of what they were, rather than what they are, and will be.

It is a pervasive criticism of school mathematics that it is impoverished in terms of connections with the lives of the students, their families, and their communities. The CREME project (see above) gave us some insights into the lives of the children we were working with. When one child was asked 'Is there any mathematics at home?' the response was only in terms of worksheets brought home from school, an image of 'what is mathematics' that we tried hard to demolish. Our attempts to involve students' families in the project were very much informed by Luis Moll's conception 'funds of knowledge' (see above).

The reality of one young person's life was brought home to us when she asked how her mother could come to the launch of the students' book since she was in prison part of what Skovsmose (e.g., 2022) refers to as her 'foreground'. Moreover, as Skovsmose (2022) points out, children's (often realistic) appraisals of their educational and life possibilities may be an important factor in their engagement or lack of engagement with the mathematics presented to them in school. To put it starkly: If a young Black person is concerned about staying out of prison or even staying alive, why should he or she be interested in solving quadratic equations?

In terms of what children are exposed to in mathematics classrooms, an issue that is discussed in Chapter 13 of this volume is that of inappropriate framing of ways in which aspects of the real world can be modelled mathematically. In particular, mathematical problems as presented may have little relevance to, and indeed be incompatible with, the experience of the students. There is no reason why children could not be introduced, from an early age, to aspects of mathematical modelling, including how to discriminate between situations precisely modelled by simple arithmetical expressions, approximate cases, and purported correspondences that are absurd.

If a test item, in a context that admits of no discussion, invokes a situation to be modelled, this can have serious implications when the interpretation of the situation is culturally or class-relative (see the discussion of the 'bus fare' problem by Tate, 1995, in Chapter 13).

The technological age

If you watch the film 'Timbuktu' (directed by Abderrahamane Sissako, 2014), you will see a young girl living in a small tent in the desert in Mali climb a hill to get reception for her cell phone to talk with her brother. These devices are everywhere and consequent changes are happening so quickly we have dangerously minimal idea of what the repercussions are. In Chapter 20 (this volume), Melissa Andrade-Molina and Alex Montecino present a striking review of the images of mathematics held, by young people in particular, as revealed by comments on social media.

As Artificial Intelligence surges, there are increasing concerns about its likely effects, many of which are already strikingly apparent. What could be more chilling than the American warmonger (and winner of the Nobel Peace Prize!) Henry Kissinger declaring the possibility of 'a world relying on machines powered by data and algorithms and ungoverned by ethical or philosophical norms' (cited by Ochigame, 2021, p. 167)? Surely this represents the ultimate dehumanisation by algorithm.

Information technological developments already have had major effects on how are lives (often without any control or access to the models) are 'formatted' by 'mathematics in action' (Skovsmose's terms). Further, Ian Hacking (1999) points to the dual aspects of phenomena and the social constructs that accrete around them, including new terms (linguistic models), and mathematical models that are developed in relation to those, resulting in 'looping effects' whereby the constructs modify the phenomena, so that 'In the end, mathematics comes to constitute basic features of our life-worlds' (Skovsmose, 2022). As analysed in Chapter 13, school mathematics manifests no recognition of these aspects of contemporary life, let alone preparing students with the appropriate dispositions and agency to respond to them.

The above are minimal comments on the harmful effects on young people of technology and the implications for their mathematical education. A very different framework is what Ron Eglash and his team term 'Ethnocomputing'. As the term suggests, it relates the representational possibilities offered by computer technology and the cultural knowledge of students.[2]

So what could be done in mathematics classrooms?

As described above in relation to the CREME project and vividly related by Schoenfeld (Chapter 14, this volume), constraints on teachers who might want to introduce something of the ethnomathematical spirit into their classes are extremely severe. We suggest that the following are minimal feasible efforts, closely related to the themes listed earlier.

Being more honest about the history of academic mathematics

It is hard to imagine that any teacher would knowingly lie to the children in their class, yet it is easy to point to ways in which mathematics teachers fail to tell the truth, insofar as it is known. In terms of the specific case of the non-mathematician Pythagoras, as discussed above, many people, including mathematicians, are misinformed. At the systemic level, the same holds for the image of academic mathematics as essentially the achievement of Europeans only, as illustrated by the quotation above from Kline.

A straightforward way in which a knowledge of the history of mathematics can enhance teaching is through the use of load-bearing examples and the examination of misattributions of pieces of mathematics (the 'Theorem of Pythagoras' being a clear example). What is called 'Pascal's Triangle' (or 'Tartaglia's Triangle' in Italy) can be shown in its Chinese version dating from the thirteenth century (see the excellent resource that is Swetz, 1994, p. 328). A short test on history of mathematics might ask: 'Which of the following was a mathematician: Pythagoras, Omar Khayyam, Lewis Carroll, Florence Nightingale' to which a reasonable answer is 'All of them, except Pythagoras'.

Another pointed exercise, for example in the contexts of statements about the importance of symbolic algebra, would be to point to

2 Rather than attempt to summarise this very comprehensively developed theory, we refer the interested reader to https://csdt.org/publications for applications to education and https://generativejustice.org/publications for applications to economy and development.

achievements in many societies in the past which pre-dated abstract algebra and calculus.

Valorising students, allowing students to valorise themselves

In the CREME project (see above), a major emphasis was on the children's images of themselves, which, from our perspective, are liable to be greatly damaged in specific ways by school mathematics, as when performance on tests is used as a proxy indication of level of intelligence. The foundations for the current extreme situation in the United States were laid with the No Child Left Behind Act of 2002. In the words of McDermott and Kathleen Hall (2007, p. 10) that presented 'a vision for achieving progress in education through increased control and standardisation, a form of rational bureaucratic authority Max Weber [...] described as central to modernity' (see Chapter 5, this volume). Absolutely central was the use of testing, with results to be reported by ethnic categories, supposedly in the service of identifying 'failing schools' that could then be helped. Instead, as McDermott and Hall (p. 11) put it: 'Quantitative tests of aptitude and achievement have given U.S. education a way to sort children by race and social class, just like the old days, but without the words "race" and "class" front and center'. All is encapsulated in the pernicious phrase 'achievement gaps', a gloss on 'differences in test scores'. These effects are magnified in mathematics, given the political importance attached to the subject and the ease with which tests can be quantified (either you can compute 3/7 + 5/9 or you can't).

There are many other factors, including, as already discussed, the ways in which images of mathematics lend themselves to the projection of intellectual White supremacy. These reasons motivated our attempts in the CREME project (see above) to valorise the abilities and knowledge of the students. At the same time, we recognise that the children are forming understandings of the obstacles facing them in contemporary socio-political circumstances that obstruct their future possibilities in life. School mathematics education plays a major role in this regard. Accordingly, we argue that access and equity to the edifice are insufficient; it is essential to go beyond, to develop agency, in particular towards critiquing the nature of the edifice.

Relating school mathematics to the lives of students, their families and communities

Parra (Chapter 10, this volume) powerfully argues for research on Ethnomathematics to be reconceptualised as *relational*. As a parallel, a great deal of what we are attempting to articulate in this chapter can be expressed as the aim of reconceptualising school mathematics as relational, above all in relation to the children's funds of knowledge.

Critical mathematics educators argue that one aspect of making school mathematics relational is to relate it to the socio-political circumstances of the students, their families and communities. A clear instantiation of this perspective lies in the work of Rico Gutstein (e.g., 2006, 2012), influenced by Paulo Freire. He describes his work with the math for social justice class of seniors in 2008–2009 in which they collectively decided on five topics (generative themes, in Freirean terms): elections, populations displacement within their city (Chicago), HIV/AIDS, criminalisation, and sexism. All of these topics lend themselves naturally to investigation using mathematics. Gutstein (personal communication, 2010) resists characterising this work as ethnomathematical, yet it is surely related to the socio-political circumstances of the students. The choice of generative themes related to their lives makes such pedagogy relational, as does another feature of his work, namely having students make presentations to the community presenting the findings of their analyses. He summarises his work as follows:

> [...] three types of interrelated, yet distinct, knowledges relate to 'reading and writing the world with mathematics' [namely] community, classical, and critical knowledges, which all have mathematical components [...] community knowledge refers to [...] knowledge of one's own life circumstances and perspectives on reality. Classical knowledg refers to 'traditional' academic knowledge and critical knowledge means critiques and analyses of relations of power and issues of (in)justice. (Gutstein, 2012, p. 27)

In Chapter 13 of this volume, the importance of the early years in the learning of mathematics is discussed. As discussed there, children learn at an early age – through absurdly unrealistic word problems, in particular – that mathematics is not expected to cohere with their lived experience, their funds of knowledge, or even to make sense. The emphasis on early imprinting of images of mathematics also applies to issues of this chapter, including the image of mathematics as the sole creation of White (male) people and the associated beliefs about who can and cannot do mathematics.

Educate the privileged and future powerful

It is natural enough that most of the writing in the spirit of this chapter has focused on improving the education of marginalised groups. However, speaking at the 2008 ICME conference in Monterrey, Mexico, Ubiratan D'Ambrosio suggested that more attention be paid to the children enjoying privilege and likely to become powerful. This echoes themes in postcolonial writing such as Albert Memmi (1957/1965) about the deadly symbiosis between colonisers and colonised. Apart from the strategic reasons for taking this suggestion seriously in terms of attempting to speak truth to power, we may also consider the intellectual and moral harm done to privileged children (compare the discussion in Chapter 18 (this volume) on 'WhiteCrit'). This is a line of argument that we hope to develop more systematically in the future.

For similar reasons, the same comment may be made about those who major in mathematics at university. There are many reasons for incorporating courses on the history of mathematics for such students, but such courses tend to be inadequate from our point of view, rarely addressing the Eurocentric bias. How many mathematicians know of the cases discussed by Rodrigo Ochigame (2021) such as the development of paraconsistent logic in Brazil in the 1950s or the idea of a non-binary Turing machine based on the Jaina sevenfold system of predication, conceived by scientists in India in the aftermath of its independence?

Final thoughts

What give mathematics and mathematics education the special character that makes it an ideological battleground? It is not controversial to talk about language as a general human faculty *and* particular languages, about *architectures*, in the plural, related to cultural and environmental diversity, about *different* forms of music. Perhaps the answer lies in the importance of mathematics as a powerful means for shaping people as desired by a state? And regardless of the nature of the regime, a common aspect of this is conformity, the following of rules.

George Lakoff (1996) proposed a way of thinking about the differences between conservatives and liberals (his words) in terms of two core metaphors, labelled 'Strict Father' and 'Nurturant Parent', and argues how these help to explain how positions in relation to so many issues that on the surface appear unrelated are, in fact, highly correlated. Foreshadowing the extremes of the contemporary situation in and beyond the United States, he wrote that:

> Conservatives have, at least since the 60s, seen their system of values under attack—from feminism, the gay rights movement, the ecological movement, the sexual revolution, multiculturalism, and many more manifestations of Nurturant Parent morality. (p. 229)

At the most general level, perhaps it is an unavoidable consequence of the complexity of modern societies that there is a conflict between education and governance (Skovsmose, 2022). Munir Fasheh asked:

> Is it possible to teach mathematics effectively – that is, to enhance a critical attitude of one's self, society, and culture; to be an instrument in changing attitudes, convictions, and perspectives; to improve the ability of students to interpret the events of their immediate community, and to serve its needs better –without being attacked by existing authorities whether they are educational, scientific, political, religious, or any other form? (Fasheh, 1982, p. 2)

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