AUGUSTUS DE MORGAN, POLYMATH

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Cover image: Portrait of Augustus De Morgan, in Sophia Elizabeth De Morgan, *Memoir of Augustus De Morgan* (*1882*), https://commons.wikimedia.org/wiki/File:Augustus_De_Morgan_1850s.jpg. Background: Nico Baum, White round light on gray textile (2020), https://unsplash.com/photos/white-round-light-on-gray-textile-xZroI5V_dxc.
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Fig. 3 De Morgan's letters, books and personal papers were often adorned with imaginative and unusual cartoons. Here, he applies his idiosyncratic sense of humour to the subject of algebra. (RAS MSS De Morgan 3, reproduced by permission of the Royal Astronomical Society Library and Archives.)

1. De Morgan and Mathematics

Adrian Rice

> De Morgan did not write mathematics;
> He wrote *about* mathematics!
>
> — J. J. Sylvester[1]

Introduction

Contemporaries of Augustus De Morgan described him as 'one of the most eminent mathematicians and logicians of his time' and 'one of the profoundest and subtlest thinkers of the nineteenth century'.[2] His mathematical publications spanned an impressive array of subjects, including algebra, logic, probability, analysis, differential equations, actuarial mathematics, mathematical education and the history of mathematics, earning him the reputation as one of Victorian Britain's most respected and influential mathematicians.

Over the years, particularly after the First World War, this reputation faded substantially. By 1935, a contributor to *The Listener* lamented: 'If a book were to appear today on the great scientists of the nineteenth century it would be safe to say you would look in vain in its index for the name of Augustus de Morgan.'[3] And although De Morgan is

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1   George Bruce Halsted, 'De Morgan to Sylvester', *The Monist*, 10 (1900), 188–97 (p. 197).

2   William Stanley Jevons, 'De Morgan, Augustus', *Encyclopædia Britannica*, 9th edn, vol. 8 (Edinburgh: Adam & Charles Black, 1877), pp. 64–67 (p. 64); Henry Enfield Roscoe, *The Life and Experiences of Sir Henry Enfield Roscoe* (London: Macmillan, 1906), p. 25. The vocabulary echoes that of De Morgan's newspaper obituaries.

3   A. S. Russell, 'Augustus De Morgan, A Forgotten Worthy', *The Listener*, 14 (24 Dec. 1935), 1161.

in fact present in the index—and the main text—of E. T. Bell's *Men of Mathematics* of 1937, it is solely for brief supporting roles in chapters on better-known contemporaneous British mathematicians.[4]

If mathematicians remember Augustus De Morgan at all today, it is primarily for the pair of algebraic laws that bear his name, for he proved no major theorem, made no notable mathematical discoveries and published no *magnum opus*. His mathematical achievements appear slim in comparison with those of his peers, such as Arthur Cayley, James Joseph Sylvester, William Rowan Hamilton and George Boole. Indeed, it is hard to identify much in today's mathematics for which De Morgan was responsible. If he is mentioned in recent historical studies of nineteenth-century mathematics, he is regarded as a minor, although charming, figure of minimal consequence to the overall development of mathematics in the period.

Such comparative indifference after seemingly universal approbation prompts the present-day reader to speculate whether his mathematical contributions are undervalued today, or, by contrast, whether such a high former reputation was deserved. This chapter will explore the range of De Morgan's activities as a mathematician to re-assess his former and his current reputation.

De Morgan's Mathematical Output

De Morgan was a voluminous writer. During a period of over forty years from the beginning of his career in 1828, he published a host of books and research papers, as well as countless unsigned articles and reviews in various journals and periodicals.[5] Some of his textbooks, originally intended for his students at University College London, sold in sufficient numbers to warrant multiple editions and translations, particularly his *Elements of Arithmetic* (1830).[6] This pioneered a new style of textbook exposition in being designed to be read by both teacher and pupil (although teachers would no doubt have been the main beneficiaries). De Morgan presented material in a clear, more user-friendly way than previous textbooks had done, beginning with

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4   E. T. Bell, *Men of Mathematics* (New York: Simon & Schuster, 1937), pp. 354, 384, 387, 434, 438, 440–41.
5   See Chapters 4 and 12 in this volume.
6   1st edn, 1830; 6th edn, 1876; Marathi translation, 1848; Mandarin translation, 1859.

simple motivating examples before introducing rules, demonstrations and more sophisticated problems. He encouraged the use of tangible objects such as pebbles, coins and counters, following Johann Heinrich Pestalozzi's advocacy of environments in which children could learn through activities and exploration, rather than the more traditional approaches.[7]

Appearing 'at a time when perhaps few teachers, as they submitted the rules of the science to their pupils, cared to establish them upon reason and demonstration', the commercial success of *Elements of Arithmetic* reflected its modest influence in the teaching of arithmetic in Britain.[8] As a reviewer in *Nature* observed in 1870: 'The effect of this work was that a rational arithmetic began to be taught generally, and the mere committing of rules to memory took its due subordinate position in the course of instruction.'[9] De Morgan's interest in mathematics education was particularly noticeable in the early 1830s, when he contributed a series of articles to the short-lived *Quarterly Journal of Education*. While his later publications on didactic matters were fewer in number, subsequent articles and book reviews in the *Educational Times* and *The Athenæum* testified to a lifelong interest in mathematical pedagogy.[10]

By the mid-1830s De Morgan's mathematical interests had broadened to include probability theory and its applications, particularly to problems in insurance. The actuarial profession was in its infancy at the time, and mathematicians with a sufficient understanding of probabilistic methods could pursue lucrative careers in the insurance business.[11] To supplement his professorial income, De Morgan served as a freelance actuarial consultant to various insurance companies and 'occupied the first place [as an actuary], though he was not directly associated with any particular office; but his opinion was sought for by professional

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7   Florence A. Yeldham, *The Teaching of Arithmetic through Four Hundred Years 1535–1935* (London: Harran, 1936).

8   Anon, 'Our Book Shelf', *Nature*, 7 July 1870, 186.

9   Anon, 'Our Book Shelf'.

10 See Chapter 6 in this volume for a discussion of De Morgan's educational writings, and also Sloan Evans Despeaux and Adrian C. Rice, 'Augustus De Morgan's Anonymous Reviews for *The Athenæum*: A Mirror of a Victorian Mathematician', *Historia Mathematica*, 43 (2016), 148–71 (pp. 156–62).

11 Interestingly, it was the pioneering work of De Morgan's great-grandfather James Dodson (c.1705–57) on mortality tables and long-term life-insurance policies that had led to the formation of the Society for Equitable Assurances on Lives and Survivorships (later known as Equitable Life) in London in 1762.

actuaries on all sides, on the more difficult questions connected with the theory of probabilities, as applied to life-contingencies'.[12] He also wrote several works on actuarial subjects. His *Essay on Probabilities and on their Application to Life Contingencies and Insurance Offices* (1838) was the first book of its kind in English and remained highly regarded in insurance literature for well over a generation.

British mathematicians had neglected the theory of probability for some time and De Morgan's work on the topic, albeit small in comparison to his output in other areas, was significant. Its stimulus was probably a need to understand the great French mathematician Pierre-Simon Laplace's seminal *Théorie Analytique des Probabilités*, which De Morgan had been asked to review, and which he regarded as 'by very much the most difficult mathematical work we have ever met with'.[13] He called it

> the Mont Blanc of mathematical analysis; but the mountain has this advantage over the book, that there are guides always ready near the former, whereas the student has been left to his own method of encountering the latter.[14]

More generally, he wrote: 'There are no questions in the whole range of applied mathematics which require such close attention, and in which it is so difficult to escape error, as those which occur in the theory of probabilities'.[15] De Morgan's book-length article on the 'Theory of Probabilities' for the *Encyclopaedia Metropolitana* constituted the first major nineteenth-century English work on probability theory. Although it contained no original results, it was a readable, if technical, elucidation of Laplace's work and, with its thoughtful simplification and clarification of many of the intricate proofs, functioned both linguistically and mathematically as a translation of Laplace's *Théorie*. It furthermore displayed a knowledge and appreciation of what we would now call mathematical statistics, devoting substantial space to data analysis, including a detailed discussion of the method of least squares.

12 Arthur Cowper Ranyard, Obituary Notice of Augustus De Morgan, *Monthly Notices of the Royal Astronomical Society*, 32 (1871–72), 112–18 (p. 116).

13 Augustus De Morgan, 'Theory of Probabilities', *Encyclopaedia Metropolitana*, 2 (London: Baldwin & Cradock, 1845), pp. 393–490 (p. 418).

14 [Augustus De Morgan], Review of *Théorie Analytique des Probabilités* (by P.-S. Laplace), *Dublin Review*, 3 (1837), 237–48, 338–54 (p. 347).

15 [Augustus De Morgan], 'Probability, Probabilities, Theory of', in *The Penny Cyclopaedia*, ed. by George Long, 27 vols. (London: C. Knight, 1833–1843), 19 (1841), pp. 24–30 (p. 29).

De Morgan thus showed himself to be one of the few mainstream British advocates of Laplacean probability during the early- to mid-nineteenth century. Moreover, by promoting its practical utility in the nascent field of insurance he helped to establish it as the basis of modern actuarial mathematics.

The early nineteenth century was marked by a concerted effort by British mathematicians to bring their work up to the standard of their more progressive European counterparts. After the unpleasant priority dispute in the early eighteenth century between Isaac Newton and Gottfried Leibniz over the invention of the calculus, British mathematicians had become considerably isolated from continental developments. This had resulted in a deterioration in the overall quality of British mathematics and a startling ignorance of contemporary mathematics overseas. Laplace's notoriously challenging five-volume *Traité de Méchanique Céleste* (1799–1825) jolted British mathematicians out of their complacency and stimulated them to learn and adopt continental techniques:

> The impact of this work on British science cannot be underestimated. It was immediately recognized by many British mathematicians as a masterpiece which crowned Newtonian mechanics and astronomy. ... The achievements of Laplace in these fields were outstanding: an urgent need arose to understand his work.[16]

As a student at Cambridge in the 1820s, De Morgan had been one of the earliest of a new generation of British mathematicians to be exposed to this renewed interest in continental mathematics. His early publications show a keen interest in, and considerable knowledge of, European (particularly French) work in several areas. One of these was the study of functions, infinite series and the theoretical underpinnings of calculus, known today as mathematical analysis.

Modern-day mathematicians more or less universally accept that the whole framework of calculus is based on the notion of a limit. Although Bernard Bolzano and Augustin-Louis Cauchy first successfully formalised this concept in the 1810s and 1820s, the idea was not yet widely accepted in the 1830s. De Morgan's *Elements of Algebra* of 1835

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16 Niccolò Guicciardini, *The Development of Newtonian Calculus in Britain 1700–1800* (Cambridge: Cambridge University Press, 1989), p. 117.

stands as the first British work to contain a formal definition of a limit, specifying: 'When, under circumstances, or by certain suppositions, we can make A as near as we please to P, ... then P is called the *limit* of A.'[17]

De Morgan followed this book with a 785-page treatise on *The Differential and Integral Calculus* (1842), in which he affirmed his adherence to the new system of Cauchy and showed a keen awareness and appreciation of the work of other European mathematicians. It covered every conceivable area of calculus and remained the most comprehensive British work on the subject for well over a generation. More than one hundred years later one of the greatest twentieth-century British analysts, G. H. Hardy, called it 'the best of the early English text-books on the calculus, contain[ing] much that is still interesting to read and difficult to find in any other book'.[18]

In addition to being the most exhaustive study of the subject to date in English, De Morgan's treatise contained a number of original results in analysis. For example, he introduced a test for the convergence of infinite series of the form

$$\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \cdots + \frac{1}{f(n)} + \frac{1}{f(n+1)} + \cdots$$

where $f(n)$ is any increasing positive function of n. By introducing the quantity

$$\rho = \lim_{n \to \infty} \frac{nf'(n)}{f(n)}$$

De Morgan determined that the series would diverge if $\rho < 1$ and converge if $\rho > 1$.[19] Moreover, if $\rho = 1$, his method contained the novel feature of an iterated procedure to determine the convergence of more problematic cases.[20]

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17   Augustus De Morgan, *The Elements of Algebra* (London: Taylor & Walton, 1835), p. 155.

18   G. H. Hardy, *Divergent Series* (Oxford: Clarendon Press, 1949), p. 19.

19   Augustus De Morgan, *The Differential and Integral Calculus* (London: Baldwin & Cradock, 1842), pp. 235–36. See also C. W. Groetsch, 'De Morgan's Series Test', *The College Mathematics Journal*, 47 (2016), 136–37.

20   De Morgan, *Differential and Integral Calculus*, pp. 324–25. An equivalent procedure was discovered independently by the French mathematician Joseph Bertrand. Consequently, the convergence test is now usually called Bertrand's Test.

De Morgan made his most progressive research contribution to this area on the subject of divergent series, one of the thorniest issues in nineteenth-century mathematics as most mainstream mathematicians categorically rejected them. As the Norwegian mathematician Niels Henrik Abel declared: 'Divergent series are the inventions of the devil, and it is a shame to base on them any proposition whatsoever. By using them, one may draw any conclusion he pleases and that is why these series have produced so many fallacies and so many paradoxes.'[21] De Morgan began his 1844 paper 'On Divergent Series':

> I believe it will be generally admitted that the heading of this paper describes the only subject yet remaining, of an elementary character, on which a serious schism exists among mathematicians as to absolute correctness or incorrectness of results.[22]

Mathematicians of the time regarded strange results like

$$1 - 1 + 1 - 1 + 1 - 1 + \cdots = \tfrac{1}{2}$$

and

$$1 + 2 + 4 + 8 + 16 + \cdots = -1$$

as meaningless and therefore useless. But they are now known to be of great value. For example, string theory in physics makes use of the peculiar formula

$$1 + 2 + 3 + 4 + 5 + \cdots = -\tfrac{1}{12}\,.$$

De Morgan cautioned strongly against rejecting such results simply through a lack of sufficient understanding: 'The history of algebra shows us that nothing is more unsound than the rejection of any method which naturally arises, on account of one or more apparently valid cases in which such method leads to erroneous results. Such cases

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21   Gaither's *Dictionary of Scientific Quotations*, ed. by Carl C. Gaither and Alma E. Cavazos-Gaither (New York: Springer, 2012), p. 2301.

22   Augustus De Morgan, 'On Divergent Series and Various Points of Analysis Connected with them', *Transactions of the Cambridge Philosophical Society*, 8 (1844), 182–203 (p. 182).

should indeed teach caution, but not rejection.'[23] Although he admitted that many divergent series were not yet sufficiently understood to be of any use, he warned: 'to say that what we cannot use no others ever can ... seems to me a departure from all rules of prudence'.[24]

Looking back over a century later, the mathematician Morris Kline candidly but fairly described De Morgan's paper as 'acute and yet confused'.[25] De Morgan made some very sound points, and his main thesis—do not reject what you do not understand—turned out to be perfectly valid. As he had written in 1840: 'I am fully satisfied that they [divergent series] have an *algebraical* truth wholly independent of arithmetical considerations: but I am also satisfied that this is the most difficult question in mathematics.'[26] But while De Morgan may have been virtually the only British mathematician of the time capable, not only of appreciating the subtleties of the problem, but also of commenting constructively on the issue, what was most remarkable for twentieth-century mathematicians like Kline and Hardy was 'that so acute a reasoner should be able to say so much that is interesting and yet to miss the essential points so completely'.[27]

In some of his related research on differential equations, De Morgan's presentation was influenced by prior work by his elder British contemporaries, Charles Babbage, John Herschel, and especially his erstwhile teacher George Peacock. Their particular technique was more algebraic in spirit than analytic, focusing principally on the clever manipulation of symbols. For example, to solve the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0,$$

they would re-write it in terms of the differential operator, D, to give

$$D^2y + Dy - 6y = 0,$$

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23   De Morgan, *Differential and Integral Calculus*, p. 566.
24   De Morgan, 'On Divergent Series', p. 183.
25   Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), p. 975.
26   Oxford, Bodleian Library, Dep. Lovelace-Byron, Box 170, Letter from Augustus De Morgan to Ada Lovelace, 15 Oct. 1840, f. 19v.
27   Hardy, *Divergent Series*, p. 20.

where $D = d/dx$. The symbols of operation would then be separated from the variable, y, to give

$$(D^2 + D - 6)y = 0 \ \text{ or } \ (D - 2)(D + 3)y = 0.$$

Now, since it was known that the solution of $(D - 2)y = 0$ is $y = c_1 e^{2x}$ and the solution of $(D + 3)y = 0$ is $y = c_2 e^{-3x}$, the solution of the original differential equation would thus be

$$y = c_1 e^{2x} + c_2 e^{-3x}.$$

De Morgan used this technique, known as the 'calculus of operations', in a paper of 1844 to analyse differential equations of the form $(D + a)^n y = X$, where a is a constant.[28] In so doing, he generated a formula in terms of the operator D^{-1}:

$$(D + a)^{-1}\{A(D + a)B\} = AB - (D + a)^{-1}(BDA).$$

Given that the function D represented the derivative, its inverse D^{-1} would naturally represent the opposite operation, namely, the integral. Thus, De Morgan's result, with $a = 0$, $A = u$ and $B = v$, was simply equivalent to the well-known formula for integration by parts, or

$$\int u \, dv = uv - \int v \, du.$$

The calculus of operations 'became a cottage industry of British mathematics in the 1840s and 1850s', featuring prominently in the work of several of De Morgan's contemporaries such as Duncan Gregory, Robert Murphy, George Boole and Arthur Cayley.[29] This algebraic inclination bore notable fruit as British mathematicians furthered the agenda initiated by Peacock's *Treatise on Algebra* (1830), in which symbolic algebra was conceived as a generalisation of the symbols and operations of basic arithmetic.

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28   Augustus De Morgan, 'On the Equation $(D + a)^n y = X$', *Cambridge Mathematical Journal*, 4 (1844), 60–62.

29   Victor J. Katz and Karen Hunger Parshall, *Taming the Unknown: A History of Algebra from Antiquity to the Early Twentieth Century* (Princeton: Princeton University Press, 2014), p. 405. See also Elaine Koppelman, 'The Calculus of Operations and the Rise of Abstract Algebra', *Archive for History of Exact Sciences*, 8 (1971–72), 155–242.

In a series of four papers, 'On the Foundation of Algebra', and the monograph *Trigonometry and Double Algebra*, De Morgan pushed algebra further towards abstraction by focusing less on the meanings of the individual symbols and more on the laws under which they operated.[30] Examples of such laws included the associative law for multiplication, which states that, for any three numbers, a, b and c,

$$a(bc) = (ab)c$$

and the commutative law,

$$ab = ba.$$

By focusing on these fundamental laws, or axioms, De Morgan helped advance a trend towards axiomatising algebra that gained pace as the nineteenth century progressed. For De Morgan, the algebraic symbols a, b and c could represent numbers, but did not have to, provided that the laws were applied correctly:

> In abandoning the meanings of symbols, we also abandon those of the words which describe them. Thus *addition* is to be, for the present, a sound void of sense. It is a mode of combination represented by +; when + receives its meaning, so also will the word *addition*. ... If any one were to assert that + and – might mean reward and punishment, and A, B, C, &c. might stand for virtues and vices, the reader might believe him, or contradict him, as he pleases—but not out of *this* chapter.[31]

At this time algebra was still largely restricted to solving equations using the basic laws of arithmetic. The most general kind of solutions were complex numbers of the form $z = a + bi$, where a and b were real numbers and $i^2 = -1$. Complex numbers could be represented geometrically in two dimensions; De Morgan termed this 'Double Algebra'. But could such a representation be extended to three-dimensional space? To answer this,

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30   Augustus De Morgan, 'On the Foundation of Algebra', *Transactions of the Cambridge Philosophical Society*, 7 (1842), 173–187; 'On the Foundation of Algebra, no. II', *Transactions of the Cambridge Philosophical Society*, 7 (1842), 287–300; 'On the Foundation of Algebra, no. III', *Transactions of the Cambridge Philosophical Society*, 8 (1849), 139–42; 'On the Foundation of Algebra, no. IV, on Triple Algebra', *Transactions of the Cambridge Philosophical Society*, 8 (1844), 241–54; *Trigonometry and Double Algebra* (London: Taylor, Walton, & Maberly, 1849).
31   Augustus De Morgan, *Trigonometry*, p. 101.

De Morgan invented a variety of so-called 'Triple Algebras' where each number had the form $z = a + bi + cj$, where $i \neq j$ and $i^2 = j^2 = -1$. But while adding such number triples together was easy, multiplication proved more of a challenge. De Morgan found that, given two triples $z_1 = a_1 + b_1 i + c_1 j$ and $z_2 = a_2 + b_2 i + c_2 j$, their product was the rather messy

$$(a_1 a_2 - b_1 b_2 - c_1 c_2) + (a_1 b_2 + b_1 a_2)i + (a_1 c_2 + c_1 a_2)j + (b_1 c_2 + c_1 b_2)ij.$$

This was clearly not a triple, so something needed to be done about the rogue ij-term at the end. Arbitrarily letting $ij = -1$, De Morgan solved this problem, but he then found that

$$i(ij) \;=\; i(-1) \;=\; -i$$

but

$$(ii)j \;=\; (i^2)j \;=\; (-1)j \;=\; -j.$$

This meant that, in general,

$$a(bc) \;\neq\; (ab)c$$

meaning that multiplication in De Morgan's triple algebra was not associative.[32] He had thus provided the first published example of what mathematicians now call a non-associative algebra.

De Morgan's research on algebra influenced William Rowan Hamilton's ground-breaking 1843 discovery of quaternions, a four-dimensional group of complex numbers, and the first known algebra to be non-commutative with regard to multiplication. Hamilton acknowledged:

> Among the circumstances which assisted to prevent me from losing sight of the general subject ... was probably the publication of Professor De Morgan's first paper on the Foundation of Algebra, of which he sent me a copy in 1841.[33]

This 'liberation' of algebra from the previously unassailable laws of arithmetic was a key development in nineteenth-century mathematics.

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32   Augustus De Morgan, 'On the Foundation of Algebra, No. IV, on Triple Algebra', *Transactions of the Cambridge Philosophical Society*, 8 (1844), 241–54 (pp. 249–51).

33   William Rowan Hamilton, *Preface to Lectures on Quaternions* (Dublin: Hodges & Smith, 1853), p. 41.

It opened the floodgates for the creation of newer and ever more unorthodox algebraic systems, including matrices, octonions and vectors. By De Morgan's death in 1871, algebra was well on its way to becoming the generalised abstract subject it remains today, with new systems of algebras being created and axioms formalised. De Morgan's contributions to the subject thus stimulated the growth of abstraction and catalysed a remarkable algebraic discovery.

As if this were not enough, De Morgan had a lifelong fascination with the history of mathematics, which he regarded as being extremely useful in mathematical research and teaching, as well as intrinsically interesting, and historical writings accounted for one-sixth of his published output.[34] In the nineteenth century he was an acknowledged authority, as renowned for his historical work as for his mathematics, and described as 'perhaps more deeply read in the philosophy and history of mathematics than any of his contemporaries'.[35] His interests were wide-ranging and his knowledge was extensive: medieval English mathematical authors, calendar reckoning, and the history of arithmetic are just a few of the diverse topics on which he published. Like William Whewell, David Brewster, and other contemporary historians of science, De Morgan in his publications on the history of mathematics generally emphasised primary and archival sources, and aimed to reconstruct as accurate a picture of past events as possible from the available evidence. But in stark contrast to his peers, De Morgan rejected the 'great man' view of history. Instead, he attempted 'to understand his mathematical predecessors not merely as intellectual forefathers but as human beings'.[36] At a time when Newton and Euclid dominated historical studies of mathematics, De Morgan believed that 'names which are now unknown to general fame are essential to a sufficient view of history'[37] and wrote:

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34 Adrian Rice, 'Augustus De Morgan: Historian of Science', *History of Science*, 34 (1996), 201–40 (p. 201).

35 Walter William Rouse Ball, *A History of the Study of Mathematics at Cambridge* (Cambridge: Cambridge University Press, 1889), p. 133.

36 Joan L. Richards, 'Augustus De Morgan, the History of Mathematics, and the Foundations of Algebra', *Isis*, 78 (1987), 7–30 (p.17).

37 Augustus De Morgan, 'The Progress of the Doctrine of the Earth's Motion between the Times of Copernicus and Galileo, being Notes on the Ante-Galilean Copernicans', *Companion to the Almanac for 1855*, 5–25 (p. 21).

> It would be much too strong a simile to compare the man whose name is in the mouths of all to the engineer who lays the match to the train, and startles the world by an explosion, while no one asks who bored the rock or laid the powder. But though such a comparison would err in degree, it will serve to remind us that, in every great achievement of human intellect of which it falls within the power of history to see the antecedents, we notice a gradual preparation, which is seldom adequately described.[38]

De Morgan is perhaps best remembered for his research into logic.[39] He published two books and a series of five papers 'On the Syllogism' between 1846 and 1863. The famous 'De Morgan's Laws' appeared in his third paper (1858). Today, they are a staple of any introductory undergraduate course in logic, where they might appear as

$$\neg(a \lor b) \equiv \neg a \land \neg b \ \text{ and } \ \neg(a \land b) \equiv \neg a \lor \neg b,$$

and in algebra, in which they could take the form

$$(A \cup B)' = A' \cap B' \ \text{ and } \ (A \cap B)' = A' \cup B'.$$

De Morgan, however, expressed them verbally: 'The contrary of an aggregate is the compound of the contraries of the aggregants: the contrary of a compound is the aggregate of the contraries of the components.'[40]

His most original contribution to the whole subject came in his fourth paper of 1860, in which he provided an analysis of the logic of relations that substantially increased the scope of the subject.[41] In particular, he categorised relations as 'identical', 'convertible' and 'transitive' (which

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38 Augustus De Morgan, 'On Some Points in the History of Arithmetic', *Companion to the Almanac for 1851*, 5–18 (p. 14).

39 See Chapter 2 of this volume.

40 Augustus De Morgan, 'On the Syllogism, No. III and on Logic in General', *Transactions of the Cambridge Philosophical Society*, 10 (1858), 173–320 (p. 208).

41 Augustus De Morgan, *On the Syllogism, and Other Logical Writings*, ed. by Peter Heath (London: Routledge & Kegan Paul, 1966), p. xx; Daniel D. Merrill, *Augustus De Morgan and the Logic of Relations* (Dordrecht: Kluwer Academic Publishers, 1992); Maria Panteki, 'French "logique" and British "logic": On the Origins of Augustus De Morgan's Early Logical Inquiries, 1805–1835', *Historia Mathematica*, 30 (2003), 278–340 (p. 280); Michael E. Hobart and Joan L. Richards, 'De Morgan's Logic', in *Handbook of the History of Logic. Volume 4: British Logic in the Nineteenth Century*, ed. by Dov M. Gabbay and John Woods (Amsterdam: North-Holland, 2008), pp. 283–329.

we would nowadays call reflexive, symmetric and transitive), and proved a variety of results about abstract relations. Today, relations are central to pure mathematics, and De Morgan's ground-breaking paper of 1860 is what initiated this important area, creating a whole new field of study for mathematicians, and making the subsequent development of subjects such as equivalence relations, adjacency relations and partially ordered sets possible.

He was also one of the few mathematicians of his time to realise the importance of logic to mathematics, and vice versa, attempting to bring mathematical ideas into his logic by introducing a numerically precise method of 'quantifying the predicate' and constructing a symbolic notation in which all reasoning could be carried out. This innovative approach directly inspired his friend George Boole to 'resume the almost forgotten thread of former enquiries' and publish his own version, ultimately leading to the creation of Boolean algebra and the birth of modern symbolic logic.[42]

De Morgan as Professor of Mathematics

In addition to being a prominent mathematical writer, De Morgan was a highly influential teacher, holding the chair of mathematics at University College London for almost his entire career. He inspired his students with a love and fascination for the subject and convinced even those who took it no further of the beauty and allure of mathematics. The historian Thomas Hodgkin recalled him from his experience as a student at UCL in the late 1840s thus:

> Towering up intellectually above all his fellows, as I now look back upon him, rises the grand form of the mathematician, Augustus De Morgan, known, I suppose to each succeeding generation of his pupils as 'Gussy'. A stout and tall figure, a stiff rather waddling walk, a high white cravat and stick-up collars in which the square chin is buried, a full but well chiselled face, very short-sighted eyes peering forth through gold-rimmed spectacles; but above all

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42   George Boole, *The Mathematical Analysis of Logic* (Cambridge: Macmillan, 1847), p. 1. See also: Luis María Laíta, 'Influences on Boole's Logic: The Controversy between William Hamilton and Augustus De Morgan', *Annals of Science*, 36 (1979), 45–65; and Anna-Sophie Heinemann, *Quantifikation des Prädikats und numerisch definiter Syllogismus. Die Kontroverse zwischen Augustus De Morgan und Sir William Hamilton: Formale Logik zwischen Algebra und Syllogistik* (Münster: Mentis, 2015).

> such a superb dome-like forehead, as could only belong to one of
> the kings of thought: that is my remembrance of De Morgan, and
> I feel in looking back upon his personality that his is one of the
> grandest figures that I have known.[43]

The lawyer James Bourne Benson affirmed that 'De Morgan [was] looked upon with awe' by the undergraduates of his day.[44] By his retirement in 1867, De Morgan's name and the quality of his instruction had established UCL as the prime source for advanced mathematical tuition in London.[45] From each of the four decades following De Morgan's 1828 inauguration as UCL's founding professor of mathematics, graduates distinguished in their chosen fields praised their former mentor fulsomely as an 'eccentric but brilliant teacher' whose lectures were stimulating, often inspiring, and far from easy.[46] Such students included the future Master of the Rolls, Herbert Cozens-Hardy; educational reformer Sir Philip Magnus; and constitutional authority Walter Bagehot.[47] Two students from the 1840s, Isaac Todhunter and E. J. Routh, spread De Morgan's pedagogical influence, going on to Cambridge, then the epicentre of mathematical education in Britain, and dominating the teaching of mathematics there for half a century: the former wrote a highly successful series of textbooks, while the latter became one of the most successful mathematical coaches in its history.

The eminent mathematician James Joseph Sylvester, famous for Sylvester's law of inertia and for introducing the word 'matrix' into mathematics, was one of De Morgan's earliest students. Although he

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43 Thomas Hodgkin, 'University College, London, Fifty Years Ago', *The Northerner*, 1 (1901), 75.

44 UCL MS (1921), Mem. 1B/3: Materials for the history of UCL: James Bourne Benson, 'Some Recollections of University College in the Sixties', f. 3.

45 Adrian Rice, 'What Makes a Great Mathematics Teacher? The Case of Augustus De Morgan', *The American Mathematical Monthly*, 106 (1999), 534–52; Adrian Rice, 'Mathematics in the Metropolis: A Survey of Victorian London', *Historia Mathematica*, 23 (1996), 376-417.

46 David Eugene Smith, *History of Mathematics* (Boston: Ginn, 1923), vol. 1, p. 462.

47 In addition to his college students, De Morgan also took private pupils, the most famous being Ada King, Countess of Lovelace and the only legitimate child of Lord Byron, who famously worked with Charles Babbage on what is widely regarded as the first computer program. De Morgan tutored Lovelace on algebra, functional equations and calculus in 1840 and 1841. See Christopher Hollings, Ursula Martin and Adrian Rice, 'The Lovelace–De Morgan Mathematical Correspondence: A Critical Re-Appraisal', *Historia Mathematica*, 44 (2017), 202–31, and *Ada Lovelace: The Making of a Computer Scientist* (Oxford: Bodleian Library Publishing, 2018).

only studied at UCL for a few months, Sylvester remained proud of his association with De Morgan, the teacher 'whose pupil I may boast to have been'.[48] The economist and logician Stanley Jevons, a student from the 1850s, described him in the *Encyclopaedia Britannica* as 'unrivalled' as a teacher of mathematics, whose 'writings, however excellent, give little idea of the perspicuity and elegance of his *viva voce* expositions, which never failed to fix the attention of all who were worthy of hearing him'.[49] The astrophysicist Arthur Cowper Ranyard corroborated:

> He had a method of interesting his hearers in the subjects on which he lectured, and of making them love mathematics for its own sake, to which few other men have ever attained.[50]

Ranyard collaborated with De Morgan's son George, himself a promising mathematician, to form the London Mathematical Society (LMS) in 1865. This body originated as little more than a mathematics club for De Morgan's current and former students, before quickly growing into Britain's *de facto* national learned society for mathematics.[51] The last major event of De Morgan's mathematical career and perhaps the most tangible by-product of his success as a teacher at UCL was his inaugural presidency of the LMS, and his opening address became the first paper to be published in the Society's *Proceedings*.

De Morgan's Philosophy of Mathematics

De Morgan was, first and foremost, a teacher.[52] Pedagogical concerns and didactic perspectives inform a significant proportion of his mathematical writings, as is apparent from his attention to epistemological issues, clarity of expression and educational utility. De Morgan wrote much of his scholarly work to be precise, intelligible and, above all, instructive, both for his own undergraduate students and for those readers who might be engaged in higher studies or mathematical research.

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48   E. T. Bell, *Men of Mathematics*, p. 384.
49   Jevons, 'De Morgan', p. 65.
50   Ranyard, *Obituary of De Morgan*, p. 115.
51   Adrian C. Rice, Robin J. Wilson, and J. Helen Gardner, 'From Student Club to National Society: The Founding of the London Mathematical Society in 1865', *Historia Mathematica*, 22 (1995), 402–21.
52   See Rice, 'What Makes a Great Mathematics Teacher?'

Admittedly De Morgan did not always achieve this aim. When laying the foundations of his subject in his inaugural lecture at UCL, for example, he did little more than appeal to idealised extensions of 'self-evident' concepts derived from experience, with striking lack of clarity for a mathematician with a reputation for logical precision:

> From the appearances of the material world, certain distinct notions are gathered, which though their prototypes have no real existence in nature, are the clearest and most definite which our minds contain. Thus, a straight line needs no definition, nor will the mention of it leave the least doubt as to what is meant in the mind of any person present.[53]

Such vagueness was typical of contemporary English scientific writers. In common with many of his countrymen, De Morgan's philosophy of mathematics was rooted in, and heavily influenced by, the English empiricist tradition of John Locke.[54] As G. H. Hardy later commented when surveying De Morgan's work in analysis: 'He talks much excellent sense, but the habits of the time are too strong for him: logician though he is, he cannot, or will not, give definitions.'[55]

De Morgan's rules for operating on his (non-defined) concepts are based on similarly empirically derived axioms, or 'necessary truths', such as 'two straight lines cannot enclose a space'.[56] Despite a largely positivist outlook, his methodology was not entirely straightforward. For example, in 1841 he was the first English mathematician to give an explicit formulation of the fundamental axioms required for an

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53   UCL Special Collections: Augustus De Morgan, 'An Introductory Lecture delivered at the Opening of the Mathematical Classes in the University of London, Novr. 5th, 1828', MS ADD 3, ff. 14–15. He was similarly vague when writing about the study of numbers: 'The first ideas of arithmetic, as well as those of other sciences, are derived from early observation. How they come into the mind it is unnecessary to inquire; nor is it possible to define what we mean by number and quantity. They are terms so simple, that is, the ideas which they stand for are so completely the first ideas of our mind, that it is impossible to find others more simple, by which we may explain them' (Augustus De Morgan, *On the Study and Difficulties of Mathematics* (London: Baldwin & Cradock, 1831), p. 4).

54   Marie-José Durand-Richard, 'Genèse de l'algèbre en Angleterre: une influence possible de J. Locke', *Revue d'Histoire des Sciences*, 43 (1990), 129–80.

55   Hardy, *Divergent Series*, p. 19.

56   [Augustus De Morgan], 'Mathematics', *The Penny Cyclopaedia*, 15 (1839), 11–14 (p. 12).

algebraic system.[57] However, having taken such an apparently modern step, he deduced nothing from these axioms. In this paper and more generally, De Morgan's mathematics, although presented as a series of logical arguments, was in reality a pragmatic blend of rigorous deductions, inductive generalisations, philosophical rumination and unsubstantiated intuition.

In her study of De Morgan's algebra, Helena Pycior divided his algebraic work into three distinct periods, or 'stages': an initial 'traditional' stage in which his algebra was grounded on self-evident first principles and motivated by allusions to real-world examples; a second 'abstract' approach, inspired by Peacock's more formal symbol-based methodology; and a final 'ambivalent' stage in which, free to invent new algebraic systems, 'he concentrated on developing a truly meaningful algebra'.[58] For this reason, she correctly pointed out that 'De Morgan's attitude towards algebra and symbolical algebra in particular changed to such an extent that it is impossible to ascribe to him any single view on the subject'.[59]

Widening this theme, Joan Richards brought in the important ingredient of De Morgan's expertise in the history of mathematics as an additional factor in the shaping of his mathematical philosophy. Just as Pycior emphasized the influence of Peacock on De Morgan's algebra, Richards also underscored the stimulus provided by Whewell with regard to the use of history as a scientific tool. De Morgan's historically inspired awareness that mathematics does not necessarily develop in a linear, orderly manner had profound consequences for his approach to the subject: 'For De Morgan the historical evolution of mathematical ideas provided important insights into the essential nature of the mathematics, which could not be counted on to fit neatly into logical or formal frameworks'.[60] It also influenced his advice for others engaged in mathematical study—as he wrote in 1859:

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57   Augustus De Morgan, 'On the Foundation of Algebra, no. II', pp. 287–88. In doing so, he unknowingly gave the first axiomatic definition of a field (minus the law of associativity) – Katz and Parshall, *Taming the Unknown*, p. 403.

58   Helena M. Pycior, 'Augustus De Morgan's Algebraic Work: The Three Stages', *Isis*, 74 (1983), 211–26 (p. 211).

59   Pycior, p. 211.

60   Richards, p. 8.

> Even in geometry and algebra, there is no method of discovery:
> the rule is, Imitate those who have succeeded, by patiently
> thinking out, as they did, the method of succeeding. You may be
> aided by observation of your predecessors: they may give useful
> hints, but not digested and infallible rules.[61]

Although De Morgan harboured reservations about certain aspects
of Whewell's philosophy, he agreed thoroughly with Whewell's
'progressive' epistemological viewpoint.[62] In a review in *The Athenæum*
in 1860, he gave the now famous example of the Four-Colour Theorem,
which he believed to rely on the principle that 'four areas cannot each
have common boundary with all the other three without inclosure'.[63]
This principle, he said, though far from obvious was not only incapable
of proof, but had also never been noticed by mathematicians before:
'Our knowledge of necessary truth, then, may be progressive'.[64]

Whewell's historically motivated study of the philosophy of science
not only convinced De Morgan of the accumulative nature of scientific
knowledge, but reinforced his conviction that an essential ingredient
for scientific (and therefore mathematical) progress is the study of its
history. As he said at the first meeting of the London Mathematical
Society in 1865:

> It is astonishing how strangely mathematicians talk of the
> Mathematics, because they do not know the history of their
> subject. By asserting what they conceive to be facts they distort
> its history in this manner. There is in the idea of every one some
> particular sequence of propositions, which he has in his own
> mind, and he imagines that that sequence exists in history; that his
> own order is the historical order in which the propositions have
> been successively evolved. The mathematician needs to know
> what the course of invention has been in the different branches of
> Mathematics ... If he be to have his own researches guided in the
> way which will best lead him to success, he must have seen the

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61 [Augustus De Morgan], Review of William Whewell's *Novum organum renovatum*,
The Athenæum, 1628 (8 January 1859), 42–44 (p. 43).

62 For a detailed discussion of De Morgan's and Whewell's contrasting 'meta-
scientific' views, see Chapter 5 in this volume.

63 [Augustus De Morgan], Review of William Whewell's *The Philosophy of Discovery*,
The Athenæum, 1694 (14 April 1860), 501–03 (p. 502).

64 [De Morgan], Review of Whewell, 1860, p. 502.

curious ways in which the lower proposition has constantly been evolved from the higher.[65]

Throughout his career De Morgan was fascinated with language and its conversion into effective symbolic notation, an extension of his conviction of the importance of precise expression in mathematics. This view also emerges in his 1865 lecture:

> If we do not attend to extension of language, we are shut in and confined by it. Of this Euclid is a good example. He was stunted by want of extension. When we come to study language in connection with Logic, we find a great many things which would hardly have been expected, and by which we may learn how we may best extend the meanings of our terms.[66]

His example was that it is not immediately obvious that the words 'of' and 'but' may be construed as logical opposites. Taking the phrases 'All of men' as meaning 'All men', and 'All but men' as 'Everything except men', he showed that since the first phrase is the opposite of the second, the words 'of' and 'but' can be seen as negations of each other. Thus, in his words, 'we begin for the first time to have a rational power of extending the meanings of words'.[67]

The theme of language was also present in his inaugural lecture of 1828. Citing Locke's *Essay Concerning Human Understanding*, De Morgan contrasted the fluidity of terminology in regular language with the relative precision of mathematical vocabulary.[68] During his career, his fascination with language intersected with his interest in, and development of, symbolic notation. Indeed, in his book-length *Treatise on the Calculus of Functions* (1836), he devoted a discursive section to the evolution of algebraic notation in which he presented the new symbolism as the outcome of an abstraction process with a long history.[69] His research into logic was also furthered by his employment of algebraic

65   Augustus De Morgan, 'Speech of Professor De Morgan, President, At the First Meeting of the Society, January 16th, 1865', *Proceedings of the London Mathematical Society*, 1st ser., 1 (1865), 1–9 (p. 6).
66   De Morgan, 'Speech', p. 8.
67   De Morgan, 'Speech', p. 8.
68   De Morgan, 'Introductory Lecture', f. 16.
69   Augustus De Morgan, *A Treatise on the Calculus of Functions* (London: Baldwin & Cradock, 1836), p. 62. [Later published in *Encyclopaedia Metropolitana*, Pure Sciences, 2 (1845), 305–92.]

symbols to facilitate logical inferences previously represented by words.[70] As he wrote, the formal manipulation of mathematical symbols should always lead to truth, but it was the linguistic interpretation of that truth that presented the greater challenge: '[T]here is every reason to hope that the symbols are always right, even though the views of their explanation may require correction.'[71]

De Morgan's philosophy of mathematics, then, was continually evolving. But three motivating features remained constant throughout: the pedagogically-inspired desire for clarity and precision; the fascination with language and notation, prompted by this need for accurate expression; and the belief that true insights into the nature of mathematics were obtainable via the study of its history.

Conclusion

Augustus De Morgan's mathematical legacy lies in four principal achievements. Firstly, his work on algebra in the 1840s was an important catalyst for the discovery of quaternions, one of the great mathematical innovations of the nineteenth century. Moreover, his work furthered the axiomatisation and abstraction of the subject, making him one of the forefathers of modern abstract algebra. Secondly, his research into logic resulted not only in the birth of modern symbolic logic and, indirectly, Boolean algebra, but also in the creation of a totally new area of mathematics, and arguably his most original contribution to the subject, the study of relations. Thirdly, he established UCL as one of the leading centres for the study of mathematics in Britain. Finally, his influential support of the London Mathematical Society at its foundation helped provide a model by which new mathematical ideas could be communicated, extended and preserved—a model that lasts to this day.

De Morgan's high mathematical reputation in the nineteenth century rested primarily on his tremendous expository skill, via both his published works and college teaching. In addition to his undergraduates, he influenced contemporaneous mathematical research. The great variety of his publication venues, from learned

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70   See Hobart and Richards, 'De Morgan's Logic'.
71   [Augustus De Morgan], 'Algebra', in *Supplement to the Penny Cyclopaedia*, ed. by George Long (London: Charles Knight, 1845), vol. 1, pp. 74–78 (p. 78).

society journals to magazines to encyclopaedias, ensured a wide and diverse audience for his writings. These ranged from textbooks and papers on mathematical pedagogy to lengthy expositions of probability and analysis that introduced sophisticated recent European methods to a British audience, and from original investigations in the history of mathematics to notable contributions to the development of both abstract algebra and mathematical logic.

Yet the very reasons for De Morgan's fame and reputation in and shortly after his lifetime simultaneously explain his relative obscurity today. A teacher's influence decays rapidly in the absence of its immediate beneficiaries. By the 1920s and 1930s, De Morgan's former students were dead, and he had ceased to be a living memory. Although he was a great mathematical writer, he was more of a supporting character than a main protagonist in ground-breaking mathematics, and more of an expositor than an originator. He was one of the most learned and erudite scholars of his time, but no single work that he wrote stands as his masterpiece. Instead, he scattered his erudition in a host of once-popular books and compendia that have become hard to find and in long-defunct esoteric journals. Consequently, the task of reading, let alone appreciating, the entirety of De Morgan's output is next to impossible, so many and varied were his areas of expertise. Perhaps the American historian of mathematics David Eugene Smith put it best when he said of De Morgan in 1923: 'Had he been able to confine himself to one line, he might have been a much greater though a less interesting man.'[72]

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72   Smith, *History of Mathematics*, p. 462.

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