AUGUSTUS DE MORGAN, POLYMATH

EDITED BY KAREN ATTAR, ADRIAN RICE AND CHRISTOPHER STRAY

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Cover image: Portrait of Augustus De Morgan, in Sophia Elizabeth De Morgan, *Memoir of Augustus De Morgan* (*1882*), https://commons.wikimedia.org/wiki/File:Augustus_De_Morgan_1850s.jpg. Background: Nico Baum, White round light on gray textile (2020), https://unsplash.com/photos/white-round-light-on-gray-textile-xZroI5V_dxc.
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Fig. 4 De Morgan's artistic flair and keen eye for design were reflected in his 'Zodiac of Syllogism', an attractive arrangement of various logical arguments exhibited in his distinctive symbolic notation, surrounding his personal monogram, which featured the letters ADM arranged in a symmetric formation. This emblem was subsequently used on the reverse of the London Mathematical Society's De Morgan Medal. (MS ADD 7, reproduced by permission of UCL Library Services, Special Collections.)

2. De Morgan and Logic

Anna-Sophie Heinemann

> Logic, the only science which is admitted to have made no improvements in century after century, is the only one which has grown no symbols.
>
> — Augustus De Morgan[1]

Introduction

Although most logicians of the present day are familiar with the propositional laws regarding conjunction, disjunction and negation that have come to bear Augustus De Morgan's name, little is known about his original work on logic. Historiographers of logic notoriously refer to him as a contemporary to George Boole, but of a more traditional mindset.[2] Clarence Irving Lewis, for example, stated in 1918 that his 'methods and symbolism ally him rather more with his predecessors than with Boole and those who follow'.[3] Eighty years later, Ivor Grattan-Guinness similarly asserted in his influential *Search for Mathematical Roots* that he 'worked largely within the syllogistic

1 Augustus De Morgan, 'On the Syllogism, No. III, and on Logic in General', *Transactions of the Cambridge Philosophical Society*, 10 (1858), 173–230 (p. 184).

2 George Boole is usually seen as the founding father of symbolic logic in a modern sense. For a critical assessment of this claim, see, for example, Volker Peckhaus, 'Was Boole Really the "Father" of Modern Logic?', in *A Boole Anthology. Recent and Classical Studies in the Logic of George Boole*, ed. by James Gasser (Dordrecht: Springer, 2000), pp. 271–85.

3 Clarence I. Lewis, *A Survey of Symbolic Logic* (Berkeley: University of California Press, 1918), p. 38.

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tradition'.[4] Authoritative assessments of this kind have rarely been questioned. Therefore, De Morgan's contributions to the logical literature of his times are not usually discussed very extensively.[5]

It is true that De Morgan's approach to logic is primordially rooted in traditional syllogistic logic. Nonetheless, it is also true that De Morgan's logic provides for certain novelties which imply some fundamental revisions of the syllogistic tradition. Apart from De Morgan's logic of relations, which has been widely recognised as a seminal contribution to the development of modern logic,[6] his theory of what he named the 'abstract copula' as an indication of relations to be defined by logical properties such as reflexivity, symmetry and transitivity should certainly be counted among those innovations.[7] The present chapter, however, will focus on how De Morgan departed from traditional syllogistic logic and

4 Ivor Grattan-Guinness, *The Search for Mathematical Roots 1870–1940. Logics, Set Theories and the Foundations of Mathematics from Cantor through Russell to Gödel* (Princeton University Press, 2000), p. 27.

5 There are of course exceptions. For example, there is an extensive discussion of De Morgan's logic in Maria Panteki's Ph.D. thesis, *Relationships between Algebra, Differential Equations and Logic in England: 1800–1860* (Ph.D. Diss., Middlesex University, London, 1991), pp. 407–92. De Morgan's logic of relations has been dealt with in Daniel D. Merrill, *Augustus De Morgan and the Logic of Relations* (Dordrecht: Springer, 1990). In 2008, Michael Hobart and Joan L. Richards published the illuminating overview 'De Morgan's Logic', in *Handbook of the History of Logic*, vol. 4, ed. by Dov Gabbay and John Woods (Amsterdam: North Holland, 2008), pp. 283–329. I myself devoted more than half of a monograph to De Morgan's logic in *Quantifikation des Prädikats und numerisch definiter Syllogismus. Die Kontroverse zwischen Augustus De Morgan und Sir William Hamilton: Formale Logik zwischen Algebra und Syllogistik* (Münster: mentis, 2015), especially pp. 105–260.

6 Again, see Merrill, *Logic of Relations*, or, for more historical context, Benjamin S. Hawkins Jr., 'De Morgan, Victorian Syllogistic and Relational Logic', *Modern Logic*, 5 (1995), 131–66.

7 De Morgan developed his notion of an 'abstract copula' in 'On the Symbols of Logic, the Theory of the Syllogism, and in Particular of the Copula, and the Application of the Theory of Probabilities to Some Questions of Evidence', *Transactions of the Cambridge Philosophical Society*, 9 (1850), 79–127 (pp. 104–14). De Morgan's abstract copula allows for logical inferences which depart from traditional syllogisms of forms such as '*S* is *M*, *M* is *P*, therefore *S* is *P*' in that they do not require a middle term (*M*) in order to derive a conclusion connecting the extremes (*S* and *P*). An example often referred to is: 'Every horse is an animal, therefore every head of a horse is a head of an animal'. The present chapter, however, will be concerned with De Morgan's modifications of syllogistic schemes which do have middle terms. While De Morgan elaborated on his logic of relations and the abstract copula from the late 1850s onwards, his modifications of syllogistic schemes with middle terms are situated in his earlier work on logic published in the 1840s and early 1850s. Throughout the present chapter, we will focus on De Morgan's logical writings from this period.

thereby revoked the notion of logical quantity of his times. We will show that De Morgan was serious about 'quantification'[8] in logic: he thought of logical quantity as resulting from an operation of enumerating members of a given set of instances of a term. In other words, De Morgan anticipated a modern sense of quantifying over a domain. However, the originality of De Morgan's stance has hardly ever been honoured.[9]

One of the reasons for this omission may lie in a certain complexity of De Morgan's writings due to which Lewis, for instance, judged De Morgan's articles 'ill-arranged and interspersed with inapposite discussion'.[10] Again, Grattan-Guinness echoed that De Morgan 'was not a clear-thinking philosopher'.[11] There are indeed some passages in De Morgan's writings which appear unclear and confusingly abundant with technical details, the productiveness of which is not always evident. The goal of the present chapter will be to sketch out De Morgan's approach to quantification without reproducing too many of De Morgan's technicalities. To this purpose, we will first summarise

8 Genetically speaking, De Morgan may have adopted the term 'quantification' from Sir William Hamilton, his opponent in the debate over the so-called quantification of the predicate. Hamilton had coined the term 'quantification of the predicate' with regard to a clarification of the propositional forms acknowledged in traditional syllogistic logic, as summarised in the second section of the present chapter. In traditional syllogistic logic, propositions are classified according to the quantity of their subject term. For example, 'All *A* is *B*' is universal as to the term *A*. Hamilton, however, would distinguish between 'All *A* is all *B*' and 'All *A* is some *B*' in order to 'quantify' the predicate term *B*. After a personal correspondence on syllogistic logic, Hamilton came to the conclusion that De Morgan had plagiarised his own thought in the paper published in 1847, which we will discuss in the main part of this chapter. The historical course of the debate between De Morgan and Hamilton is outlined in Peter Heath, 'Editor's Introduction', in *Augustus De Morgan: On the Syllogism and Other Logical Writings*, ed. by Peter Heath (London: Routledge & Kegan Paul, 1966), pp. vii–xxxi (pp. xi–xxiv). An overview is also given in Luis María Laíta, 'Influences on Boole's Logic: The Controversy between William Hamilton and Augustus De Morgan', *Annals of Science*, 36 (1979), 45–65 (pp. 51–60). A detailed reconstruction can be found in my *Quantifikation*, pp. 23–58.

9 Daniel Bonevac, 'A History of Quantification', in *Handbook of the History of Logic*, vol. 11, ed. by Dov Gabbay, John Woods and Francis J. Pelletier (Amsterdam: North Holland, 2012), pp. 63–126, for example, omits reference to De Morgan altogether, except for a casual remark stating that De Morgan adopted Hamilton's scheme of quantification (p. 94). A closer look both at De Morgan's and at Hamilton's writings, however, would have revealed that this cannot possibly be the case. I have tried to substantiate this claim in *Quantifikation*, especially pp. 21–22, 39–41, 52–58.

10 Lewis, *Survey*, p. 38, fn. 1.

11 Grattan-Guinness, *Mathematical Roots*, p. 27.

some basic traits of traditional syllogistic logic as a point of departure. Subsequently, we will explain some of De Morgan's modifications by reference to two of his systems of syllogistic inference.

Point of Departure: Traditional Syllogistic Logic

Traditional accounts of syllogistic logic admit of four propositional forms.[12] These are compounds of a subject term, *S*, and a predicate term, *P*, to be distinguished by a quantitative specification of the subject term, as indicated by 'all' or 'some', and by a qualitative specification of the copula, to be expressed by 'is' or 'is not'. For short, the letters *A*, *E*, *I* and *O* stand for

A: All *S* is *P*,

E: All *S* is not *P* (i.e. No *S* is *P*),

I: Some *S* is *P*,

O: Some *S* is not *P*.

As a propositional form, *A* is universal and affirmative, *E* is universal and negative, *I* is particular and affirmative, and *O* is particular and

12 For the purposes of the present chapter, the name 'syllogistic logic' should be taken to denote a version of Aristotelian logic handed down to nineteenth-century Britain through early modern writers, most prominently Henry Aldrich. His *Artis Logicae Compendium*, first published in 1691, saw multiple editions and translations into English, as well as abridged and annotated versions for the use of schools up to the year 1900. Other early modern authors to be named are Edward Brerewood, Richard Crackanthorpe, Robert Sanderson, John Wallis and Isaac Watts. Around the middle of the 1820s, a significant revival of interest in syllogistic logic was prompted by Richard Whately's article for the *Encyclopaedia Metropolitana*, first published as a monograph in 1826: *Elements of Logic. Comprising the Substance of the Article in the Encyclopaedia Metropolitana; with Additions, &c.* (London: Mawman, 1826). The effects of this revival are discussed in Chapter 5 of this volume. An overview of early nineteenth century British logic is given in James W. Allard, 'Early Nineteenth-Century Logic', in *The Oxford Handbook of British Philosophy in the Nineteenth Century*, ed. by W. J. Mander (Oxford: Oxford University Press, 2014), pp. 25–43. Calvin Lee Jongsma discussed Whately's role in his Ph.D. dissertation, *Richard Whately and the Revival of Syllogistic Logic in Great Britain in the Early Nineteenth Century* (Ph.D. Diss., University of Toronto, 1982).
The account in this section is based on an 1821 abridged and annotated edition, which is likely to mirror the standard logic of De Morgan's times: Henry Aldrich, *Artis Logicae Rudimenta from the Text of Aldrich. With Illustrative Observations on Each Section*, 2nd edn (Oxford: Baxter, 1821).

negative.[13] In other words, each of the four letters is used to sum up two specifications, each of which may be of two kinds: A proposition may be assigned universal or particular quantity, while it may be of affirmative or negative quality.

On the traditional account, a proposition's quantity is determined according to the quantity of the subject term alone.[14] A proposition of the form *A*, for example, is universal because it states that all of the subject term *S* belongs to *P*, the predicate. A proposition of the form *E* is also universal since it states that all of *S* does not belong to *P*, i.e., that none of *S* belongs to *P*. In both cases, the subject term is said to be distributed,[15] which means that the proposition makes a claim about every member of the class denoted by the subject term. In the case of *I* and *O*, not all *S*, but only some of *S* is stated to belong to *P* or not to belong to *P*, respectively. Accordingly, *I*- and *O*-propositions are not universal, but particular, and their subject terms are not distributed.[16]

For the purposes of traditional syllogistic logic, no explicit mention of the quantity of *P* is necessary. In the case of affirmative propositions *A* and *I*, it is to be understood that *S* belongs to *P*, but does not necessarily exhaust it. For example, all humans are mortal, but it is not the case that all mortals are human. However, it may be that all humans are rational animals and all rational animals are humans. Therefore, the quantity of the predicate remains indefinite in the sense of being unspecified. Hence in terms of traditional syllogistic logic, *P* is not distributed in affirmative propositions. Negative propositions *E* and *O*, however, imply that *S* does not belong to *P*. Since this means that *S* is apart from all of *P*, the quantity of the predicate is definite. Accordingly, in negative propositions, *P* is always distributed.[17] In short:

(i) In propositions of the form *A*, only the subject term is distributed.

(ii) In propositions of the form *E*, both the subject term and the predicate term are distributed.

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13 Aldrich, *Rudimenta*, p. 86.
14 Aldrich, *Rudimenta*, p. 79.
15 Aldrich, *Rudimenta*, p. 86.
16 Aldrich, *Rudimenta*, p. 86.
17 Aldrich, *Rudimenta*, pp. 86–87.

(iii) In propositions of the form *I*, neither the subject nor the predicate term is distributed.

(iv) In propositions of the form *O*, only the predicate term is distributed.

These rules have certain implications for the validity of syllogisms. A syllogism is defined as a combination of two propositions serving as premises such that a third proposition, the conclusion, is to be inferred. One common scheme is '*S* is *M*, *M* is *P*, therefore *S* is *P*'.

Syllogistic inferences, then, are possible if and only if the premises share one term, the so-called middle term (*M*), in a way allowing for a specification of the relation between the remaining two components. These are usually called the 'extremes' (*S* and *P*). But according to the traditional account, connecting the extremes is not possible if the middle term is not distributed in at least one of the premises,[18] or if both premises are negative,[19] or if both premises are particular.[20] In other words, the basic guidelines of traditional syllogistic logic are:

(I) The middle term must be distributed in at least one of the premises.

(II) At least one of the premises must be affirmative.

(III) At most one of the premises must be particular.

In the remainder of this chapter, we will discuss how De Morgan's approach to logic departs from (I), (II) and (III) just given. We will point out that the reason De Morgan's logic allows for such departures lies in

18 Aldrich, *Rudimenta*, pp. 121–22.

19 Aldrich, *Rudimenta*, p. 125.

20 Aldrich, *Rudimenta*, p. 125. The quoted edition of Aldrich has twelve rules altogether. They can be summarised as follows: 1.) A syllogism must involve three terms. 2.) A syllogism must consist of three propositions. 3.) The middle term must not be ambiguous. 4.) If the middle term is not distributed, no conclusion is possible. 5.) The middle term must be distributed in at least one of the premises to allow for a conclusion. 6.) If one of the other terms is not distributed in the premises, it cannot be distributed in the conclusion. 7.) If both premises are negative, no conclusion is possible. 8.) If one premise is negative, the conclusion must be negative. 9.) If the conclusion is negative, it must be that one of the premises is negative. 10.) If both premises are particular, no conclusion is possible. 11.) If one of the premises is particular, the conclusion must be particular. 12.) If the conclusion is particular, it is not the case that one of the premises is necessarily particular. (Aldrich, *Rudimenta*, pp. 116–132).

the fact that he dismissed the traditional understanding of propositions as codified in rules (i), (ii), (iii) and (iv).

Syllogistic Logic in the 'Language of Numeration of Instances'

According to De Morgan, his logical writings speak a 'language of numeration of instances'.[21] This means that De Morgan's logic is not so much about conceptual spheres, i.e., the meanings of the terms chosen as subject and as predicate, but about sets of instances denoted by these terms. In other words, the relations of inclusion and exclusion between subject terms and predicate terms are to be interpreted extensionally. They pertain to sets of instances of terms, portions of which may map onto each other if there are pairwise coincidences of certain instances of each set. Coincidences of this kind lie in that a given member of a set is an instance both of the subject and the predicate term. For example, each particular member of the set of individuals denoted by the term 'human' is at the same time a member of the set of individuals denoted by the term 'mortal'. In other words, the very same individual is human and mortal at the same time. However, there are instances of the term 'mortal' which do not map onto any of the instances of the term 'human'.

It is a substantial consequence of De Morgan's approach that propositions of the traditional forms of *A*, *E*, *I* and *O* may be re-stated by reference to the complements of those portions of sets of instances of terms which are referred to in the original statement. But reference to complements requires a counting of the instances included and those excluded in a term's extension. Of course, this requirement cannot be met in principle if the total number of instances is indefinite. Therefore, the notion of a term's being definite or distributed must be reconsidered. As we will see, De Morgan's reconsiderations allow for revisions of the rules (i), (ii), (iii) and (iv) of traditional syllogistic logic, as summarised in the previous section of this chapter. Consequently, the basic guidelines (I), (II) and (III) become negotiable.

In what follows, we will address De Morgan's revisions of traditional syllogistic logic in his 'system of contraries' and his 'numerically definite

21 De Morgan, 'Symbols', p. 96.

system'.[22] De Morgan spelled out his system of contraries in his first substantial paper on logic, 'On the Structure of the Syllogism', published in 1847.[23] He re-stated it in a more systematic fashion in the body of 'On the Symbols of Logic', published in 1850.[24] The numerically definite system was first suggested in an 'Addition' to De Morgan's 'Syllogism' paper of 1847.[25] De Morgan discussed it at length in his monograph on *Formal Logic*, published in 1847.[26]

Notably, the system of contraries dispenses with rule (I), i.e. that the middle term must be distributed in at least one of the premises. At least hypothetically, it also undermines rule (II), i.e., that one premise at least must be affirmative. The numerically definite system additionally revokes rule (III), i.e., that the premises may not both be particular.

The 'System of Contraries': Terms and Contraries

A predicate, De Morgan said in 1847, is basically a 'term' or 'name' which should be understood as a 'word' which may legitimately be applied to

22 De Morgan introduced both terms retrospectively in 1850 ('Symbols', p. 101, p. 102 and p. 79, respectively.)

23 Augustus De Morgan, 'On the Structure of the Syllogism, and on the Application of the Theory of Probabilities to Questions of Argument and Authority', *Transactions of the Cambridge Philosophical Society*, 8 (1847), 379–408. The paper was included in the volume of the Transactions for 1847, which, however, did not appear until 1849 (as can be seen in Chapter 12 of this volume). It consists of two parts, the main text and an 'Addition' (pp. 406–08). The main text is dated 3 October 1846, and it was read before the Society on 9 November. The 'Addition', however, is dated 27 February 1847. In the course of the De Morgan-Hamilton debate on the 'quantification of the predicate' (see fn. 8), Hamilton tried to substantiate his claim that De Morgan had plagiarised his own innovations in the time elapsed between acceptance of the paper by the Society and the submission of the 'Addition'. However, Hamilton apparently did so without having seen any of the two parts: He accused De Morgan of plagiarism in a letter dated 13 March 1847, while on 27 March he confirmed that he had not yet received the preprints which De Morgan had announced to him on 16 March. The correspondence is reproduced in William Hamilton, *A Letter to Augustus De Morgan, Esq. Of Trinity College, Cambridge, Professor of Mathematics in University College, London, on His Claim to an Independent Re-Discovery of a New Principle in the Theory of Syllogism. Subjoined, the Whole Previous Correspondence, an A Postscript in Answer to Professor De Morgan's 'Statement'* (Edinburgh: Maclachlan & Stewart, 1847), p. 26.

24 It was only in 1850 that De Morgan referred to it by the name of 'system of contraries' ('Symbols', p. 101, p. 102).

25 De Morgan, 'Structure', pp. 406–08.

26 Augustus De Morgan, *Formal Logic, or: The Calculus of Inference, Necessary and Probable* (London: Taylor & Walton, 1847), pp. 141–70. Again, De Morgan introduced the name 'numerically definite system' only in 1850 ('Symbols', p. 79).

any instance in a 'collection of objects of thought'. As a general rule, attributions of this kind are encoded in affirmative propositions of the form '*S* is *P*', or, as De Morgan preferred to put it, '*X* is *Y*'. However, according to De Morgan, it is not the case that in negative propositions of the kind '*X* is not *Y*', *Y* is to be taken as the predicate, connected to *X* via a negative copula. On De Morgan's account, '*X* is not *Y*' should instead be read as '*X* is non-*Y*', the predicate 'non-*Y*' being affirmatively connected to the subject term. Hence 'non-*Y*' may be taken as the complement of *Y*, or in De Morgan's terminology, *Y*'s 'contrary', i.e., '*y*'.[27]

At first glance, the versed logician might object that attributing *y* to any subject will be contradictory, not contrary to attributing *Y*. To be contradictory would mean that *y* should refer to everything that is not *Y*, to the effect that an attribution of *y* and an attribution of *Y* could neither be true nor false at the same time. Like a pair of contradictory assertions, two contrary attributions cannot both be true. However, it is possible that both are false at the same time. De Morgan thought of contraries as opposed to given terms within a restricted frame of reference. He labelled the frame of reference in question the 'universe of a proposition, or of a name'.[28] Taken together, a term and its contrary are apt to exhaust the given universe. In De Morgan's words, 'every thing in the universe is either *X* or *x*'.[29] But on a larger scale, there may be things which belong neither to *X* nor *x*. De Morgan's example is that if the universe is that of humans (or citizens), 'Briton' and 'foreigner' are contrary to each other.[30] There is no question of stones, trees, books and the like, which, viewed on absolute terms, are of course also non-Britons, but just as well non-foreigners. Within the given universe, however, foreigners may be referred to as non-British, or Britons as non-foreigners. Hence in relation to a given universe, terms may count as each other's complements without producing contradictory assertions on an absolute scale.

The 'System of Contraries': Propositions

It is obvious from De Morgan's example just quoted that context determines which is to be taken as the positive term and which as the

27 De Morgan, 'Structure', p. 379.
28 De Morgan, 'Structure', p. 380.
29 De Morgan, 'Structure', p. 380.
30 De Morgan, 'Structure', p. 380.

negative, i.e., the contrary. But therefore, assertions about each of them will be re-statable by reference to the other. This is why De Morgan claimed that as soon as contraries are systematically considered, distinctions between affirmative and negative as well as between universal and particular propositions turn out to be 'accidents of language, at least for logical purposes'.[31] For it is possible, as De Morgan suggested, that an expression which denotes a certain set of objects may be rendered in another language only as a negative correlative of another, while a third language may provide no name for the whole set of objects in question at all.[32] But therefore, translations of assertions from the first into the second language would require an apparent change of a proposition's quality. Translations from one of them into the third language, however, would call for particular propositions where the first and the second employ universals. However, according to De Morgan, this does not imply a difference as to logical structure and import. It is in this sense that he arrived at the conclusion that

> in truth, every proposition distributes, wholly or partially, among the individuals of the predicate, or of its contrary; making one particular or universal, according as the other is universal or particular.[33]

For example, since all humans are mortal, but not all mortals are human, the proposition 'all humans are mortal' distributes partially among the individuals denoted by the term 'mortal'. On the other hand, it is evident that all humans are excluded from the contrary term 'non-mortal'. Therefore, the proposition 'all humans are not non-mortals' distributes wholly among the individuals of the contrary of 'mortal'. Moreover, De Morgan's account seems to imply that 'all humans are mortal, but not all mortals are human' implies that there are some non-humans which are not mortal. It is in this sense that he explained in 1850: 'Again, "Every *X* is *Y*" denies of some *x*s that they are *Y*s: for *Y*s must not fill the universe.' Similarly, he claimed that by '"some *X*s are *Y*s" I deny something of every *x*: namely, that any one of them is one of those *Y*s'.[34]

31 De Morgan, 'Structure', p. 380.
32 De Morgan, 'Structure', p. 380.
33 De Morgan, 'Structure', p. 382.
34 De Morgan, 'Symbols', p. 92.

Consequently, a given proposition can always be transformed into an equivalent expression if on substitution of terms by their contraries, correlated variations of quantity and quality are taken into account. A catalogue of transformation rules was first suggested in De Morgan's 1847 'Syllogism' paper.[35] An extended version was offered in De Morgan's 1850 article.[36] We will return to De Morgan's transformation rules in the following subsection, as soon as a short exposition of De Morgan's notational systems has been given with a special eye to the relativity of quantity and quality.

Table 2.1 includes both versions of De Morgan's notation for the four traditional forms of *A*, *E*, *I* and *O*, interpreted extensionally.

Table 2.1 De Morgan's notational systems.

Traditional syllogistic logic	De Morgan's notation of 1847[37]	Interpretation as of 1847[38]	De Morgan's notation of 1850[39]	Interpretation as of 1850[40]
A	*X*)*Y*	Every *X* is *Y*.	*X*))*Y*	Every *X* is [some] *Y*.
E	*X* . *Y*	No *X* is *Y*.	*X*) . (*Y*	No *X* is [any] *Y*.
I	*XY*	Some *X* is *Y*.	*X*( )*Y*	Some *X*s are [some] *Y*s.
O	*X*:*Y*	Some *X* is not *Y*.	*X*(. (*Y*	Some *X*s are not [any] *Y*s.

The interpretations given for the 1847 version correspond to De Morgan's own. The interpretations given for the 1850 notation, however, add quantifiers for the predicate term. This addition to the verbal circumscription is warranted by De Morgan's systematic use of parentheses in his notational system: according to De Morgan's paper of 1850, universal quantity is indicated by a bracket to suggest a circle around the term sign such that it 'would be inclosed if the oval

35 De Morgan, 'Structure', p. 381.
36 De Morgan, 'Symbols', p. 91.
37 De Morgan, 'Structure', p. 381.
38 De Morgan, 'Structure', p. 381.
39 De Morgan, 'Symbols', p. 91.
40 De Morgan, 'Symbols', p. 91, quantifiers for the predicate term added.

were completed',[41] as in '$X)$'. Accordingly, the bracket in '$X($' might be interpreted as the remnant of an intersection of two circles which cuts out a portion of X's scope.

It is evident that De Morgan's notation for propositions has symbolic quantifiers for both the subject and the predicate term. If we remind ourselves that interpretations must conform to De Morgan's principle of making his logical systems speak a 'language of numeration of instances',[42] more detailed ways of verbal circumscription suggest themselves. De Morgan's A could be read as 'for every member of the set of objects denoted by X, there is a member of the set of objects denoted by Y'. His E would be 'for every member of the set of objects denoted by X, there is not a member of the set of objects denoted by Y, or, X and Y denote mutually exclusive sets of objects'. De Morgan's I could be interpreted as 'for at least one of the members of the set of objects denoted by X, there is a member of the set of objects denoted by Y'. Finally, De Morgan's O would be 'for at least one of the members of the set of objects denoted by X, there is not a member of the set of objects denoted by Y'.

We can now see more clearly the way De Morgan conceived of the relations between subject terms and predicates as overlaps between sets of instances of terms, i.e., of objects. The following quote gives evidence that these relations imply quantification in the sense of enumeration:

> The Xs being distinguished as X_1, X_2, X_3 *&c.*, the universal "Every X is Y" affirms that X_1 is Y, and that X_2 is Y, and that X_3 is Y, *et caetera* [while] the particular "some Xs are not Ys" only declares that either X_1 is not Y, or that X_2 is not Y, or that X_3 is not Y, *aut caetera*.[43]

In other words, 'the universal speaks conjunctively, the particular disjunctively, of the same set'.[44] Again, it should be emphasised that on De Morgan's account, this goes for both the subject and the predicate term. An affirmative universal proposition, for example, speaks conjunctively of its subject term, but it speaks disjunctively of its predicate since, traditionally speaking, the latter is not distributed.

41 De Morgan, 'Symbols', p. 86.
42 De Morgan, 'Symbols', p. 96.
43 De Morgan, 'Symbols', pp. 81–82.
44 De Morgan, 'Symbols', p. 81.

In 1847, De Morgan makes the four forms indicated in the table yield eight variants altogether: $X)Y$, $Y)X$, $X\,.\,Y$, $Y\,.\,X$, XY, YX, $X{:}Y$ and $Y{:}X$. The eight variants reduce to six since according to De Morgan, $X\,.\,Y$ is equivalent to $Y\,.\,X$ and XY is equivalent to YX as to their logical import. However, on De Morgan's account, substitution of terms by contraries additionally yields $x)y$, $x\,.\,y$, xy, $x{:}y$.[45] De Morgan took $x)y$ to be equivalent to the conversion of $X)Y$, i.e., to $Y)X$; similarly, he held that $x{:}y$ is equivalent to a converted $X{:}Y$, i.e., $Y{:}X$. However, $x\,.\,y$ and xy seem to have no equivalents. For short, De Morgan labelled these forms *e* and *i*, respectively. Following De Morgan's own interpretation, *i* states that *X* and *Y* are not contraries and therefore do not exhaust a given universe. In other words, there are objects in the universe which are neither *X* nor *Y*. But, according to De Morgan, *e* is the negation of *xy*. Hence in De Morgan's interpretation, *e* asserts that it is false that there are objects in the universe which are neither *X* nor *Y*. However, it does not preclude that there are objects which are both.[46] As an interpretation of De Morgan's 1850 notation, *i* might be read as 'some non-*X*s are some non-*Y*s', or, 'for at least one member of the set that is the complement of all instances of *X*, there is at least one member of the set that is the complement of all instances of *Y*'. For *e*, however, the interpretation could be 'all non-*X*s are not among any of the non-*Y*s', or, 'the complement of all instances of *X* and the complement of all instances of *Y* are mutually exclusive'.

Table 2.2 indicates De Morgan's full inventory of 'fundamental propositions'. Again, both versions of De Morgan's notation are compared.

45 De Morgan, 'Structure', p. 382. It is unclear why in 1847, De Morgan did not consider inverted variants in the case of contraries. Granting that $y\,.\,x$ and yx are dispensable in the sense of being equivalent to $x\,.\,y$ and xy, one should still assume that on systematic variation, $x)y$ and $y{:}x$ should also be taken into account. The exposition which De Morgan offered in 1850 is much more systematic in this respect.

46 De Morgan, 'Structure', p. 382.

Table 2.2: De Morgan's fundamental propositions.

Notation of 1847[47]		Notation of 1850[48]
A	$X)Y$	$X))Y$
a	$Y)X = x)y$	$x))y$
E	$X\,.\,Y$	$X)\,.\,(Y$
e	$x\,.\,y$	$x)\,.\,(y$
I	XY	$X(\;)Y$
i	xy	$x(\;)y$
O	$X{:}Y$	$X(\,.\,(Y$
o	$Y{:}X = x{:}y$	$x(\,.\,(y$

Clearly, the fundamentals of De Morgan's logic go beyond traditional syllogistic logic in providing twice as many propositional forms. But moreover, we should remind ourselves that on De Morgan's account, any negative proposition should be capable of being transformed into an affirmative on substitution of terms by contraries. Table 2.3 contains a systematic catalogue of transformations:[49]

Table 2.3 Transformations in De Morgan's system of contraries.

Notation of 1850
$X))Y = X)\,.\,(y = x((y = x(\,.\,)Y$
$x))y = x)\,.\,(Y = X((Y = X(\,.\,)y$
$X)\,.\,(Y = X))y = x(\,.\,)Y = x((Y$
$x)\,.\,(y = x))Y = X(\,.\,)Y = X((y$
$X()Y = X(\,.\,(y = x)(y = x)\,.\,)Y$

47 De Morgan, 'Structure', p. 381.
48 De Morgan, 'Symbols', p. 91.
49 De Morgan did not himself develop the full catalogue (cf. 'Symbols', p. 91). He did, however, give transformation rules which allow for its completion. I have tried for a more detailed discussion of these rules in '"Horrent with Mysterious Spiculae": Augustus De Morgan's Logic Notation of 1850 as a "Calculus of Opposite Relations"', *History and Philosophy of Logic*, 39 (2018), 29–52.

$x()y = x(.(Y = X)(Y = X).)y$
$X(.(Y = X()y = x).)y = x)(Y$
$x(.(y = x()Y = X)(y = X).)Y$

The 'System of Contraries': Syllogisms

Departing from his fundamental propositions, De Morgan claimed to derive all forms of syllogistic inference which are valid on the traditional account. However, De Morgan's extended syllogistic logic also provides for two forms of inference which cannot be accounted for in the traditional system. Notably, they involve the new variants *e* and *i*, as introduced above.

In his first 'Syllogism' paper, De Morgan claimed to have derived the inference schemes i_{AA} and I_{ee}, i.e., a syllogism which infers an *i*-conclusion from two affirmative universal premises, and a syllogism which infers an affirmative particular conclusion from two *e*-premises. Both violate the rules of traditional syllogistic logic as summarised in the second section of the present chapter.

In i_{AA}, both premises are affirmative and universal at least as to their subject term. In terms of traditional syllogistic logic, their subject terms are distributed. However, according to presumption (i), their predicate terms cannot be. The reason is that in affirmative propositions, nothing is said about whether the predicate term is exhausted by the subject term (extensionally or intensionally). Granting that the middle term would be in predicate position (as in 'All *S* is *M*, all *P* is *M*, therefore ...'), no conclusion would be possible since it is not distributed in either of the premises and nothing can be said about an intersection between *S* and *P* via *M* if rule (I) holds. For example, if all men are cheese-eaters and all mice are cheese-eaters, no conclusion can be drawn about a relationship between men and mice if the traditional account is granted. De Morgan, however, derived a conclusion which does not make a statement about an intersection between *S* and *P*, but about how the complements of *S* and *P* may relate to each other since *i* refers to contraries only.

To contextualise our example, we may quote from De Morgan's correspondence with Sir William Hamilton. 'This is an old trap for a beginner', De Morgan said,

> A man eats cheese,
> A mouse eats cheese,
> Therefore…
> The beginner who falls into the trap says, "a man is a mouse," and his teacher shows him, as he thinks, that no inference can be drawn. But there is an inference, namely, that there are things which are neither men nor mice, namely all which do not eat cheese.[50]

Of course, the only way of making sense of this example is to presuppose that De Morgan had in mind a generic interpretation of 'a man' and 'a mouse'—and that vegan lifestyles had not yet been invented for humans (nor for mice). Granting these limitations, however, i_{AA} is an exception to rule (I) inasmuch as it allows for the middle term not being distributed.

In I_{ee}, a particular affirmative conclusion is inferred from two negative universals. Again, traditional syllogistic logic precludes inferences from two negative premises according to rule (II). In De Morgan's case, however, both premises do not concern the terms of the conclusion, but their contraries. In other words, the premises speak of the complements of what is denoted by the terms that the conclusion speaks of. The case of I_{ee} is a bit less perspicuous than i_{AA}. Adapting the example quoted above, the premises could look like

1. All non-humans are not among any of the non-cheese-eaters (i.e., the set of non-humans and the set of non-cheese-eaters are mutually exclusive).
2. All non-mice are not among any of the non-cheese-eaters (i.e., the set of non-mice and the set of non-cheese-eaters are mutually exclusive).

Of course, it would be blatantly false to conclude 'Some humans are mice'. But it seems that this is due to empirical, not logical reasons. Remember that on De Morgan's account, *e* states that it is false that there are objects in the universe which are neither *X* nor *Y* but does not preclude that there are objects which are both.[51] In other words, *e* leaves open if the terms whose contraries it speaks of are themselves contrary to each other. Hence the terms whose contraries are connected in an *e*-proposition do not necessarily exhaust the universe in question.

50 Quoted in Hamilton, *A Letter to Augustus De Morgan*, p. 23.
51 De Morgan, 'Structure', p. 382.

Accordingly, it must remain an open question whether the sets denoted by the terms are indeed disjoint. Therefore, there is at least a hypothetical conclusion to the possibility of an overlap. In the case at issue, this would mean that the universe of cheese-eaters could include more than men and mice and that a separate criterion would be required to test whether the set of men and the set of mice are mutually exclusive. Hence I_{ee} at least hypothetically undermines rule (II) since it allows for the premises both being negative.

The 'Numerically Definite System'

The previous section served to show how De Morgan introduced two novel inference schemes which violate the most prominent guidelines of traditional syllogistic logic as summarised in the second section: On the one hand, i_{AA} is an exception to rule (I) inasmuch as it allows for the middle term not being distributed. On the other, I_{ee} at least hypothetically undermines rule (II) since it allows for the premises both being negative. However, there is a third principle not yet touched upon in De Morgan's system of contraries, namely rule (III), which precludes inferences from two particular premises. In an 'Addition' to his first 'Syllogism' paper,[52] however, De Morgan outlined some considerations that imply the very possibility of dispensing with rule (III). He then extended upon these considerations in *Formal Logic*, published in the same year.

De Morgan's numerically definite system shares one essential presupposition with his system of contraries, namely that any term or contrary 'distributes among the individuals' which it denotes. However, within the context of a proposition, it may do so 'wholly or partially'.[53] Accordingly, a universal proposition such as 'Every *X* is *Y*', De Morgan said, 'is distributively true, when by "Every *X*" we mean each one *X*: so that the proposition is "The first *X* is *Y*, and the second *X* is *Y*, and the third *X* is *Y*, &c."'[54]

This approach conforms to De Morgan's principle that the instances contained in a given universe must be countable at least in principle. De Morgan's extension presently discussed, however, requires that they

52 De Morgan, 'Structure', pp. 406–408.
53 De Morgan, 'Structure', p. 382.
54 De Morgan, *Formal Logic*, p. 144.

must be numerically specified. A particular proposition such as 'Some *X*s are *Y*s' should then be spelled out as 'Every one of *a* specified *X*s is one or other of *b* specified *Y*s.' A negative particular, on the other hand, would read 'No one of *a* specified *X*s is any of *b* specified *Y*s'.[55]

This approach implies a sense of predication—and therefore, of logical inference—as based on one-to-one-mappings of instances of terms. Therefore, De Morgan labelled it the 'numerically definite system',[56] as based on the notion of 'definite particulars'.[57] Numerically definite inferences, then, should be derivable from premises such as 'if there be 100 *Y*s and we can say that each of 50 *X*s is one or other of 80 *Y*s, and that no one of 20 *Z*s is any one of 60 *Y*s'.[58]

In fact, it is not immediately evident where premises of this kind lead to. De Morgan's *Formal Logic* provides a very detailed technical apparatus including case-by-case analyses for specified numbers of *X*s being greater or smaller than the specified numbers of *Y*s or *Z*s. However, for reasons of both space and clarity, we will omit further references to these discussions here. Nevertheless, there is one aspect of De Morgan's numerically definite system that we will take up for the very reason that it facilitates dispensing with rule (III) of traditional syllogistic logic, as stated in the second section of the present chapter. This aspect is De Morgan's 'ultratotal quantification' of the middle term. Maybe its clearest statement in *Formal Logic* is as follows:

> We cannot show that *X*s are *Z*s by comparison of both with a third name, unless we can assign a number of instances of that third name, *more than filled up* by *X*s and *Z*s: that is to say, such that the very least number of *X*s and *Z*s which it can contain are together more in number than there are separate places to put them in. ... Accordingly, so many *X*s at least must be *Z*s as there are units in the number by which the *X*s and *Z*s to be placed, together exceed the number of places for them.[59]

In this context, De Morgan distinguished two combinations of premises that allow for inferences, namely a combination of two affirmative

55 De Morgan, 'Structure', p. 406.
56 De Morgan, 'Symbols', p. 79.
57 De Morgan, *Formal Logic*, p. 15.
58 De Morgan, 'Structure', p. 406.
59 De Morgan, *Formal Logic*, p. 154.

propositions on the one hand, and a combination of one affirmative and one negative proposition on the other. In both cases, Y serves as the middle term and a limited universe is given. The first combination is that

1. A specified number m out of a total of ξXs maps onto the same number of instances of Y.
2. A specified number n out of a total of ηYs maps onto the same number of instances of Z.

The second case is a combination of one affirmative and one negative premise, namely that

1. A specified number m out of a total of ξXs maps onto the same number of instances of Y.
2. A specified number n out of a total of ζZs does not map onto a specified number s out of the total of ηYs.

Hence according to De Morgan, given

ξ: total number of Xs,
η: total number of Ys,
ζ: total number of Zs,
ν: total number of instances in the universe,[60]

the combinations of premises are

$$mXY + nYZ,$$
$$mXY + nZ : sY.$$[61]

In the first case, neither m nor n exceed the total number of instances of the middle term, η. In other words, both the specified number of Xs and the specified number of Zs each fall short of the total number of Ys. However, if their conjunction does exceed the total number η of Ys, it is possible to infer that there is an overlap between the specified scope of X and the specified scope of Z. This overlap must then contain as many elements as lie between η and the sum of m and n:

60 De Morgan, *Formal Logic*, pp. 143–44.
61 De Morgan, *Formal Logic*, p. 145.

$$mXY + nYZ = (m + n - \eta)\, XZ.$$ [62]

For the second combination of premises, De Morgan considered a case-by-case-analysis for the sum of m and s exceeding η on the one hand, and the sum of n and s exceeding η on the other. According to De Morgan, if the sum of m and s is larger than η, the excess elements do not map onto any of the nZs. In case n and s exceed η, the conclusion is that the specified mXs do not map onto any of the excess elements:

$$\text{for } m + s > \eta, \qquad mXY + nZ{:}sY = (m + s - \eta)X{:}nZ$$
$$\text{for } n + s > \eta, \qquad mXY + nZ{:}sY = mX{:}(n + s - \eta)Z.$$ [63]

Since in all the cases just discussed, the numbers m, n and s are specified selections out of a total number, they may all be classified as inferences from particular premises in which the middle term is not distributed amongst the total number of individuals denoted by it. Therefore, we may infer that De Morgan's numerically definite system is apt to allow for conclusions both from two particular premises and from pairs of premises that lack a distributed middle. Hence they undermine the guidelines (I) and (III) of traditional syllogistic logic, as given in the second section of this chapter. However, note that the numerically definite system does not allow for inferences from two negative premises, which means that it does not dispense with rule (II).

62 De Morgan, *Formal Logic*, p. 145. The thought suggests itself that the size of the overlap, which depends on the numerically specified extensions of the terms, determines how probable it is for one particular individual denoted by X to be identical with an instance of Z. It is interesting that much earlier than engaging with logic, De Morgan went beyond pure mathematics in publishing *An Essay on Probabilities, and Their Application to Life Contingencies and Insurance Offices* (London: Longman, Brown, Green & Longmans, 1838). A related hypothesis would be that De Morgan's numerically definite approach in syllogistic logic stems from his involvement with applied probability. Section V of his first 'Syllogism' paper as well as Chapters IX and X of *Formal Logic* point in a similar direction. For a discussion of relations between De Morgan's stances in probability and in syllogistic logic, see Adrian Rice, '"Everybody Makes Errors": The Intersection of De Morgan's Logic and Probability, 1837–1847', *History and Philosophy of Logic*, 24 (2003), 289–305, especially pp. 293–96.

63 De Morgan, *Formal Logic*, pp. 145–46. De Morgan himself did not make use of $>$. Therefore, we add the case-by-case-analysis above according to his verbal descriptions.

Summary

In the course of the present chapter, we have endeavoured to give an overview of De Morgan's early thought on logic. We pointed out that it is rooted in the syllogistic tradition handed down to nineteenth-century Britain through various editions of early modern works. Unlike De Morgan's logic of relations and his notion of an abstract copula, his syllogistic logic never did turn out to be particularly trendsetting. As mentioned in our introduction, his attempts at casting syllogistic logic into a more technical form are usually regarded as inferior, especially when compared to the achievements of George Boole. However, we have tried to show that De Morgan's syllogistic logic does provide for certain novelties and that they relate to his approach to quantification.

In his first 'Syllogism' paper, De Morgan introduced the notion of 'contraries' of terms within a given 'universe'. According to this, a term and its contrary exhaust the given universe. This implies that the instances denoted by each of them are countable at least in principle. The same holds for the total number of instances contained in the universe. Granting these assumptions, De Morgan arrived at the conclusion that assertions about terms and contraries turn out to be re-statable by reference to the other if on substitution of terms by their contraries, correlated variations of propositions' quantity and quality are taken into account. On this basis, De Morgan introduced two novel inference schemes which violate the principles of traditional syllogistic logic, as summarised in our second section: The scheme i_{AA} is an exception to rule (I) inasmuch as it allows for the middle term not being distributed. The scheme $I_{ee'}$, however, at least hypothetically undermines rule (II) since it allows for the premises both being negative. In short, De Morgan's 'system of contraries' dispenses with the guidelines (I) and (II), but keeps rule (III), which demands a distributed middle term.

In an 'Addition' to his 'Syllogism' paper and in his monograph on *Formal Logic*, however, De Morgan developed a 'numerically definite' system, which allows for inferences from pairs of premises lacking a distributed middle. It requires that the instances of terms within a given universe must not only be countable in principle, but numerically specified. If this requirement is met, the system allows for violations of guidelines (I) and (III), but it keeps rule (II), according to which no inferences can be drawn from two negative premises.

De Morgan's numerically definite system appears to be consistent with his claim that all works on logic speak a 'language of numeration of instances'. However, while some of its specific assumptions were items of controversy even in his own times,[64] logicians of the present day usually judge it a dead-end in the history of modern formal logic.[65]

We may conclude that even if De Morgan's early logical systems have not themselves been very influential, both give evidence of De Morgan's sense of logical quantification, which amounts to conjunction or disjunction of definite or at least specifiable numbers of instances both in universal and particular cases. It is this sense of quantification which has survived in modern formal logic when it comes to quantifying over domains.

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64 See, for example, James Broun, 'The Supreme Logical Formule [sic]', *The Athenæum*, 1025, 19 June 1847, 645–646. De Morgan himself did not elaborate on the numerically definite system in his later writings. There is, however, a paper by Boole entitled 'On Propositions Numerically Definite', which was posthumously read to the Cambridge Philosophical Society by De Morgan in 1868 and published in 1871 (George Boole, 'On Propositions Numerically Definite. By the late George Boole, F.R.S., Professor of Mathematics in Queen's College. Communicated by A. De Morgan, Esq.', *Transactions of the Cambridge Philosophical Society*, 11 (1871), 396–411).

65 Ian Pratt-Hartmann, for example, holds that 'no finite collection of syllogism-like rules, broadly conceived, is sound and complete for the numerical syllogistic' ('No Syllogisms for the Numerical Syllogism', in *Languages: From Formal to Natural. Essays Dedicated to Nissim Francez on the Occasion of his 65th Birthday*, ed. by Orna Grumberg, Michael Kaminski et al. (Berlin, Heidelberg: Springer, 2009), pp. 192–203 (p. 192); cf. 'The Syllogistic With Unity', *Journal of Philosophical Logic*, 42 (2013), 391–407 (p. 391).

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