AUGUSTUS
DE MORGAN,
POLYMATH



EDITED BY KAREN ATTAR, ADRIAN RICE AND CHRISTOPHER STRAY



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Cover image: Portrait of Augustus De Morgan, in Sophia Elizabeth De Morgan, *Memoir of Augustus De Morgan* (1882), https://commons.wikimedia.org/wiki/File:Augustus_De_Morgan_1850s.jpg. Background: Nico Baum, White round light on gray textile (2020), https://unsplash.com/photos/white-round-light-on-gray-textile-xZroI5V_dxc. Cover design: Jeevanjot Kaur Nagpal.



Fig. 9 A student's sketch of De Morgan teaching at University College London in 1865. (MS ADD 7, reproduced by permission of UCL Library Services, Special Collections.)

6. De Morgan and Mathematics Education

Christopher Stray

The experience of every day makes it evident that education develops faculties which would otherwise never have manifested their existence.

Augustus De Morgan¹

ugustus De Morgan is well known as a writer on mathematics and logic, as a historian of mathematics, and as a teacher of the subject at University College London for several decades. In this chapter, I focus on a relatively unstudied aspect of his work: his writing on the contemporary teaching and learning of mathematics.² In this he drew on his own education at school and, especially, at Cambridge (1823–27), but also on his own career at the University of London (1828–31), renamed University College London (1836–66), and on his knowledge of teaching at Oxford and other institutions. The basis of his assessment of educational practices was analysis and comparison in both time and space. Spatial comparison was made between London, Cambridge, Oxford and sometimes Paris; temporal comparison involved his own teaching and his own past learning. As he put it with characteristic pointedness, when he gave his introductory lecture at the University of London on 5 November 1828, 'I began to teach myself to better purpose

¹ Augustus De Morgan, On the Study and Difficulties of Mathematics (London: Baldwin & Cradock, 1831), p. 3.

² My thanks for helpful discussion to Karen Attar, Nicolas Bell, Jonathan Smith and especially Adrian Rice.

than I had been taught, as does every man who is not a fool, [when he begins to teach others,] let his former teachers be what they may'.³

Of the seven schools De Morgan attended as a child, the only one he found of any use was that run by Revd John Parsons in Redland near Bristol, which he attended from midsummer 1820 to Christmas 1822.⁴ Parsons was an Oxford graduate; though his 1805 BA was only a pass degree, he was later elected to a fellowship at Oriel College (1807–12).⁵ As in most schools in that period, Classics received much more attention than mathematics.⁶ De Morgan's account of his experiences there indicates both the dominance of Classics and his objection to the rote learning typical of the period:

The poor ignorant Virgil and Homer scanners, and their subordinate Euclid and algebra drillers, had not the slightest idea that *a memory* is the adjunct of each faculty, that the training of one is of little or no help to another, and that the memory of words, which they over-cultivated, differs widely among young people. The allowance was forty lines a day, Latin and Greek alternately, for five days in a week, the whole to be repeated in one lot on Saturday.⁷

Why 'subordinate'? Because the education of the middle and upper classes in this period was dominated by Classics, beginning with the rote learning of composition and translation, and the mechanical learning of grammar, syntax and verse-making. This last is what is referred to by 'scanners': scanning the metre of Latin and Greek poetry, then attempting to reproduce it in 'nonsense verses' (written with no regard for sense), then 'sense verses', which made sense as well as scanning metrically.⁸ This classically-dominated scene included the public schools and

Quoted by Sophia Elizabeth De Morgan, Memoir of Augustus De Morgan (London: Longmans, Green, 1882), p. 29, from his 'A True and Authentic List of the Teachers of A. De Morgan', in his 'Memorandums on the Descendants of Captain John De Morgan...', UCL Special Collections, MS. ADD. 7, f. 155. Sophia omitted the phrase in parentheses from her published memoir.

⁴ S.E. De Morgan, *Memoir*, pp. 5–9; UCL MS Add 7, f. 154.

⁵ Parsons was given the living of Marden, Wiltshire in 1816, and apparently kept it till his death on 31 July 1844 (*Gentlemans Magazine*, 176 (1844), p. 327).

⁶ In the 1850s, De Morgan learned from Robert Leslie Ellis (Trinity 1836, Senior Wrangler 1840) that Ellis's two brothers had also been at Parsons's school (S.E. De Morgan, *Memoir*, p. 9). De Morgan and Ellis were alike in that they eventually concentrated on mathematics, while retaining a knowledge of Classics; and both men had claims to polymathy.

⁷ S.E. De Morgan, Memoir, p. 8.

Oxford; Cambridge stood out as an anomaly because of the centrality of mathematics to its curriculum, the Senate House Examination being the university's only degree examination. The Cambridge colleges, however, were homes of classical teaching and learning, though some of them, notably St John's, until the 1790s the largest college in the university, were important centres of mathematical study. Boys coming up to Cambridge were first admitted to a college and then matriculated (enrolled) at the university. The annual college examinations formed part of a transition from the classical curriculum of most schools to the mathematical content of the Senate House examination, which was taken in an undergraduate's tenth term.

This transition can be seen in De Morgan's progress through Trinity.¹¹ He was admitted to the college on 1 February 1823, and so entered in a by-term: in other words, not in the Michaelmas (autumn) term which began the academic year. In the college examination in May he was top of the second class. In her memoir, his widow Sophia De Morgan pointed out that this was only three months after his admission, but we should also take account of the fact that mathematics was a minority presence in this examination. His mother was keen for him to concentrate on Classics rather than mathematics, then to become a clergyman in the evangelical wing of the Church of England.¹² In 1824, however, when he was a 'junior sophister' (second-year undergraduate), he was in the first class in the second-year examination, which was dominated by

⁸ Two of the lower forms at Eton were named after this system, Nonsense and Sense. Cf. C.A. Stray, *Classics Transformed: Schools, Universities, and Society in England 1830–1960* (Oxford: Oxford University Press, 1998), pp. 68–71. Some schoolmasters rose above mere scanning: in 1820 De Morgan's headmaster had read out one hundred lines of Homer versified by Walter Scott, as De Morgan recalled in a letter to John Herschel of 29 April 1862 (S.E. De Morgan, *Memoir*, p. 309).

⁹ The examination became known as the Mathematical Tripos after the foundation of the Classical Tripos, first examined in 1824.

The college examinations had been instituted first at St John's in 1765, Trinity following suit in 1790. By 1830 they were in place at all the colleges. Cf. C.A. Stray, 'From Oral to Written Examination: Oxford, Cambridge and Dublin 1700-1914', History of Universities, 20 (2005), 76–130.

¹¹ The best treatment of this period in De Morgan's life remains A.C. Rice, 'Augustus De Morgan and the Development of University Mathematics in London in the Nineteenth Century' (Unpublished Ph.D. Diss., Middlesex University, 1997), pp. 20–35.

¹² S.E. De Morgan, *Memoir*, p. 12. He came under considerable pressure from his mother to attend the sermons of Charles Simeon, the evangelical vicar of Holy Trinity church, rather than just to 'the [college] chapel' (S.E. De Morgan, *Memoir*, p. 13).

mathematics. His tutor, John Higman, told De Morgan's mother that he was 'not only in our first class, but far, very far, the first in it'. ¹³

By this time he had, according to Sophia, been captivated by George Peacock's mathematical lectures, which opened up 'new life' for him as he listened to 'Peacock's explanations'. Sophia refers to these as 'university lectures', but such lectures were in this period given only by professors, who usually charged for attendance; Peacock was not a professor until his election to the Lowndean chair of astronomy in 1837. He had been appointed an assistant tutor on being elected fellow of Trinity in 1814, so would have been giving lectures within one of the three tutorial sides (in this period undergraduates attended lectures only within their own sides). In 1823 he was appointed a tutor in succession to John Hustler, so had probably taught on Hustler's side earlier; De Morgan was perhaps given permission to attend his lectures. De Morgan's own tutor John Higman seems to have been the strongest influence on him; he was one of those who sent in testimonials for De Morgan's successful application for the London mathematics chair in 1828 (in fact he sent two), while Peacock did not send a testimonial.14

According to Sophia, De Morgan lived out of college for the first two years. ¹⁵ This was a common experience for Trinity undergraduates until a new court was opened in 1824, planned to cope with rising numbers of students and also to protect them from the temptations of extracollegiate life. ¹⁶ De Morgan would normally have gone on to his third year (senior sophister) in 1824-5, but appears to have degraded, in effect taking the year off. ¹⁷ This practice became quite common in the 1820s, perhaps in response to the increasing difficulty of the Mathematical Tripos; in effect, it gave an undergraduate an additional year to prepare

¹³ S.E. De Morgan, Memoir, p. 12.

¹⁴ See Adrian Rice, 'Inspiration or Desperation? Augustus De Morgan's Appointment to the Chair of Mathematics at London University in 1828', *British Journal for the History of Science*, 30(3) (1997), 257–74.

¹⁵ S.E. De Morgan, *Memoir*, pp. 12–13.

¹⁶ The court was to be named King's Court or Brunswick Court, but in the end was simply called New Court. The building of the new court prompted the recording of room rents; De Morgan first appears in Easter 1826, living in Q1 Great Court, where he stayed until Lent term 1827. Sophia refers to his living 'over the gate', but Q staircase is in fact next to the Queen's Gate, on the south side of Great Court, the Great Gate being on the east side.

¹⁷ Sophia mentioned that he had been ill, perhaps as a result of overwork (*Memoir*, p. 15).

for the examination.¹⁸ In De Morgan's case, it made sense to gain extra time after his truncated freshman year. He went on to win a college scholarship; Sophia's memoir places this in April 1825, but the college records make it clear that it was a year later.¹⁹ He sat the Tripos in January 1827, emerging as Fourth Wrangler. Like William Whewell before him, he was defeated by men who were inferior mathematicians but superior examination candidates; this was perhaps the original source of the attacks on the domination of education by examinations that are such a feature of his later critiques of Cambridge mathematical training.

De Morgan also enjoyed music, being a talented flautist. On one occasion, musical performance combined with academic reading. As he wrote to William Rowan Hamilton in 1858:

When I was an undergraduate, it happened to me to get very jolly in company with a party who were celebrating the new scholarship of our host. Being, as aforesaid, merry, we proceeded to sing; when it struck one of our party that we could sing as well as the choristers, a notion which came of punch and not of reason. To test the point we all got our surplices, and stood round the table, when a question arose as to what we should chant. Some one proposed $PV.VG:QV^2::CP^2:DC^2$, which met with approbation. We tried to make it fit all manner of tunes; I remember 'Zitti Zitti', 'the Evening Hymn', and 'The Campbells are coming'. But we left off with a notion that Newton was not so easily set to music as we thought.²⁰

¹⁸ This led in 1829 to a regulation that in order to check the practice, those who degraded after 30 Oct. 1830 could not obtain scholarships or sit for mathematical honours without special permission. Grace of 27 Feb. 1829, Cambridge University Archives, Degr.13.26. The term 'degrade' was glossed in a contemporary dictionary of Cambridge slang, *Gradus ad Cantabrigiam* (London: J. Hearne, 1824), pp. 43–44. The practice was disapproved of by some: in a review of J.M.F. Wright's *Alma Mater*, a memoir of his time at Cambridge 1813-19, Henry Southern referred to Wright's having *degraded* – that is, of his having descended from a struggle with his equals, to contend with the men of the year below. 'Alma Mater, or Seven Years at Cambridge', *London Magazine*, 7 (1 April 1827), 441–54 (p. 454).

¹⁹ *Memoir*, p. 15, followed by Rice, 'Augustus De Morgan', p. 32. Of the friends of about his own age mentioned in the *Memoir* (p. 16), one gained a scholarship in 1824, two in 1825.

²⁰ De Morgan to Hamilton, 1 April 1858 (see Robert Perceval Graves, *Life of Sir William Rowan Hamilton* (Dublin: Hodges, Figgis, 1889), vol. 3, p. 546). This event can be dated to April 1824 or 1825. The proposed text was from Book I, Proposition X of the *Principia*: the theorem in which Newton derives the Inverse Square Law for elliptical orbits. In 1818 the exam subjects for the second year were changed; the new list was headed by *Principia* 1, 6-14. De Morgan's notebook on *Principia* 1.10-11 survives and is dated Sept. 1824: Senate House Library, University

'Zitti Zitti' was a famous trio from Act 2 of Rossini's *Barber of Seville* (1816); Charles Wesley's Evening Hymn was published in his *Hymns and Sacred Poems* (1740); 'The Campbells are coming' was a Scots folk tune dating from around 1715. Altogether, this was a splendid crossover event, whose bizarre nature is echoed in a letter De Morgan received in 1831 from W.H. Smyth, secretary of the Royal Astronomical Society, who remarked that 'the disputations system, being both irritable and irritating, is altogether as unsuitable for astronomers as would be the dramatising of Newton's Principia'.²¹

Most of De Morgan's writing on education dealt with university teaching in Oxford, Cambridge or London. University College was his home institution, in the sense that his whole teaching career was spent there, but Cambridge was his alma mater, and much of his writing on education consisted of a critical commentary on Cambridge from the vantage point of London. But if we compare the two universities, we need to remember that the London University, founded in 1826, took a substantial number of its staff from Cambridge, and that this complicates the task of comparison.²² The first professors of mathematics, Latin and Greek at the London University, as it then was (De Morgan, Thomas Hewitt Key and George Long) were all alumni of Trinity College, Cambridge, as was Long's successor Henry Malden.²³ Key and Long had been founding professors at Jefferson's new University of Virginia before coming to London; in both places it was possible to avoid the religious restrictions of Anglican institutions like Oxford and Cambridge, where college fellowships were vacated on marriage and

of London, Add MS 775/338. This is one of 17 surviving notebooks; most are undated, but they can be assigned to the period from summer 1824 to the end of 1826 (A.C. Rice, 'Augustus De Morgan', p. 381).

²¹ S.E. De Morgan, *Memoir*, p. 44. Disputations, held in Latin, were a relic of the medieval oral examination system. They were used to pre-sort tripos candidates, but became clumsy and inefficient, and were abolished in 1839.

²² This is a case of what has been called 'Galton's Problem': comparing entities which interact. See Raoul Naroll, 'Galton's Problem: The Logic of Cross Cultural Research', Social Research, 32 (1965), 428–51.

²³ The historian of University College devoted to these four a section entitled 'A Cambridge group': H.H. Bellot, *University College, London* 1826-1926 (London: University of London Press, 1929), pp. 80–96. Key held the chair of mathematics in Virginia, changed to Latin when he reached London, and moved to a chair of comparative grammar when he became headmaster of the University's junior department, University College School.

renewed only on ordination. At Oxford the curriculum was dominated by Classics, mathematics being a minority subject with small numbers of both teachers and entrance scholarships; in Cambridge, the dominant subject was mathematics, its primacy not challenged by the Natural Sciences Tripos, founded in 1851, until after De Morgan's death twenty years later.²⁴

De Morgan's focus on the Cambridge system was due to several factors: his own experience of it, his continuing contact with Cambridge men like Whewell and Peacock, and the fact that Cambridge was the dominant source of advanced mathematical teaching in Britain. De Morgan was uniquely placed to assess Cambridge mathematics, since his undergraduate career had been located at a time when Newtonian fluxions were giving way to the analytical/algebraic approach promoted by Peacock, Charles Babbage and John Herschel. De Morgan wrote to Whewell in 1861:

Thank heavens I was at Cambridge at the interval between two systems, when thought about both was the order of the day even among undergraduates. There are pairs of men alive who did each other more good by discussing \dot{x} versus dx, and Newton versus Laplace, than all the private tutors ever do.²⁵

This transitional state is reflected in De Morgan's undergraduate notebook on the differential calculus, where he noted that there were

Different Systems pursued – Leibnitz used differentials of descending orders. Newton used the Principle of Limits or Fluxions. Lagrange rejecting both infinitesimals and Limits has used a method purely algebraical.²⁶

De Morgan outlined his negative views of the Cambridge system on numerous occasions throughout his career. Several of his reviews for

²⁴ For the history of the Mathematical Tripos, see J.W.L. Glaisher, 'The Mathematical Tripos', Proceedings of the London Mathematical Society, 18 (1886-7), 4–38 and W.W. Rouse Ball, A History of the Study of Mathematics at Cambridge (Cambridge: Cambridge University Press, 1889), pp. 187-219. The outstanding modern account is Andrew Warwick, Masters of Theory: Cambridge and the Rise of Mathematical Physics (Chicago: University of Chicago Press, 2003).

²⁵ De Morgan to Whewell, 20 Jan. 1861 (S.E. De Morgan, Memoir, p. 306).

²⁶ Senate House Library, University of London, MS 775/332, f. 1: De Morgan notebook, [1823–24]. Cf. Rice, 'De Morgan', p. 32.

The Athenæum pursued this theme: for example, in an 1856 review of a volume containing a collection of examination papers, he opined that '[a]t Cambridge subjects are *got up* to be *written out*; by many they are *crammed*, by some they are understood'.²⁷

In De Morgan's eyes, the fundamental flaw in the system of examinations employed at Cambridge and elsewhere was that they instilled the necessity for the student to 'cram' knowledge by means of hasty and unsystematic revision. Consequently, a candidate for such an examination

employs himself in collecting, without an attempt to digest. He puts by his unfinished and half-learnt material, to await the time when the examination is close at hand. Then, in the few days or weeks which precede the trial, he makes a rush at his crude mass of ill-understood notes, and endeavours to charge his unfortunate memory with the whole of it. There is no time to think of a process, to disentangle a confusion, or to give invention a fair chance of suggesting something for future consideration. All that is wanted is, to show a mass of learning on the day of examination, to make one successful effort during a few hours.²⁸

Later the same year, in an *Athenæum* review of an 1848 pamphlet proposing reforms at Oxford and Cambridge, De Morgan indicted both universities for the inadequate teaching which had prompted the rise of private tuition. This had led

not merely to the indolent student having the dose of *cram*, to speak the slang of the seats of learning, which will just serve him for his trial, but to the candidate for honours being fed on a diet which, though there may be larger allowance of it, is often not a bit more wholesome ... There is nourishment at both Universities in plenty for those who will take it in proper quantities and allow it natural digestion; and there are many who do so take it—but they are certainly not the majority. ... Whatever of sound culture may be proposed, there are many who will neither read nor think except for the examination which is to them all in all.²⁹

^{27 [}De Morgan], '[Review of] The examination papers of the Society of Arts, June 1856', The Athenæum, 1520 (13 Dec. 1856), p. 1531.

²⁸ De Morgan, 'On the Effects of Competitory Examinations, Employed as Instruments in Education', *The Athenæum*, 1096 (28 Oct. 1848), pp. 1076–77; repr. in *The Educational Times* (1 Dec. 1848), pp. 56–59.

^{29 [}De Morgan], Review of C. Daubeny, Brief Remarks on the Correlations of the Natural Sciences, The Athenæum, 1070 (29 April 1848), p. 431. Charles Daubeny was

De Morgan's feelings were shared by another Trinity man, Robert Leslie Ellis (Senior Wrangler 1840). Ellis hated the obsession with competition, which could in a sense be avoided only by coming first. In 1839 he wrote of 'my ... repugnance to the wrangler making process. There is but one place for me, & that I cannot obtain.³⁰

The obsession with intense competition was not the only aspect of Cambridge mathematics of which De Morgan complained. The way in which teaching and research were organised meant that it was almost impossible for teachers and students outside Cambridge to access the publications associated with the Mathematical Tripos. This problem was identified by De Morgan in a review of George Peacock's Algebra published in 1835.31 As he pointed out, Cambridge students focused on Cambridge tripos problems rather than on textbooks, and so the whole system was both intense and introverted.³² The scale of the problem was reflected in a satirical pamphlet written by Duncan Farquharson Gregory, Fifth Wrangler of 1837, in 1838, entitled a 'Prospectus of the Society for the Translation of Cambridge Mathematical Books into Intelligible English'.33 This opened by explaining that 'from the singularity of dialect which prevails among the Works published at this University ... these works are wholly unintelligible to people at large unconnected with Cambridge'.³⁴ The principal objects of the Society are then listed:

Professor of Chemistry at Oxford.

³⁰ Cambridge, Trinity College Library, Add. MS a.82.1: R.L. Ellis, diary 8 Feb.1839. Cf. C.A. Stray, 'From Bath to Cambridge: The Early Life and Education of Robert Leslie Ellis', in A Prodigy of Universal Genius: Robert Leslie Ellis 1817–1859, ed. by L.M. Verburgt (Berlin: Springer Nature, 2022), pp. 3–19.

³¹ De Morgan, 'Peacock's Algebra', Quarterly Journal of Education, 9 (1835), 293–311.

³² De Morgan, 'Peacock's Algebra', p. 299. The implications of this situation were well discussed by Andrew Warwick in his *Masters of Theory*, pp. 151–54.

³³ Anonymous, but ascribed in MS to Gregory and dated 21 November 1838. Copies in Cambridge University Archives, CUR 28.6.2, and in Cambridge University Library, Cam a.500.9.22; repr. in C. Whibley, In Cap and Gown: Three Centuries of Cambridge Wit (London: Kegan Paul, 1889), pp. 161–64. Gregory was the founding editor of the Cambridge Mathematical Journal (1837-), on which he worked with Archibald Smith (Senior Wrangler 1836) and Robert Leslie Ellis (Senior Wrangler 1840).

³⁴ De Morgan had made the same point: '... the Cambridge works are found difficult by other students, except those who know the secret'. (De Morgan, 'Peacock's Algebra', p. 299.)

I To translate the letter-press into the English language ... the Society will not consider itself restricted to the ordinary proportion of one line of explanation to seven pages of symbols.

...

III Where possible, to discover and explain the author's meaning in those passages where he does not seem to have fully comprehended it himself.

The Cambridge system took some of its intensity from the fact that examiners were often chosen from very recent high-performing graduates. In 1865, after having watched and commented on Cambridge mathematics for several decades, De Morgan remarked that 'The Cambridge examination is nothing but a hard trial ... between the Senior Wrangler that is to be this present January, and the Senior Wranglers of some three or four years ago'. These young examiners had a great degree of freedom in setting examination papers, and this at times led to considerable variety in the content, style and mathematical notation of questions, a variety which caused difficulty to examination candidates. In 1835, De Morgan complained:

To have no community of system—to have the moderators of one college using one, and those of another using another—each forcing his own upon the whole University during his year of office,—to oblige the same student to read books of different notation ...—to keep him in suspense as to what notation he will be examined in ... will be no advantage to the cause of science in Cambridge.³⁶

³⁵ De Morgan, 'Speech of Professor De Morgan, President, At the First Meeting of the Society, January 16th, 1865', Proceedings of the London Mathematical Society, 1 (1865), 1-9 (pp. 3–4); cf. S.E. De Morgan, Memoir, p. 283. In 1869 an article on wranglers stated that 'As a general thing, the year after their MA degree finds them and other high wranglers setting Senate-house problems and riders in their turn'. (Anon., 'A Word about Wranglers', Daily News, 30 Jan. 1869, p. 5.)

³⁶ De Morgan, 'Cambridge Differential Notation. On the Notation of the Differential Calculus, Adopted in Some Works Lately Published in Cambridge', *Quarterly Journal of Education*, 8 (1834), 100–10 (p. 110). This was largely a critique of Samuel Earnshaw's *On the Notation of the Differential Calculus* (Cambridge: J. and J.J. Deighton, 1832). Earnshaw had been Senior Wrangler in 1831 and was a successful coach from then till 1847. For an analysis of the problems with the Mathematical Tripos in the 1830s and 1840s, see C.A. Stray, 'The Slaughter of 1841: Mathematics and Classics in Early Victorian Cambridge', *History of Universities*, 32:2 (2022), 143–78.

This lack of central control had its advantages, as De Morgan recognised, in that a single examiner could introduce an innovation via the papers he set, as George Peacock had in 1817 when he employed analytic notation.³⁷ In 1848 an examining board was set up to deal with this issue of control and consistency; it issued annual reports and recommendations, and presided over an examination whose structure remained largely unchanged until 1873.³⁸

In the following year, De Morgan sent a typically witty comment on Cambridge teaching to Whewell:

The spoon will do well for inductive logic ... What is more inductive than a spoon?—moreover, spoon feeding is synthetical, which induction is—knife and fork feeding is analytical ... remember that the spoon process, as hitherto understood at Cambridge is rather ingoosative than inductive.

This letter plays on the connotation of force feeding in 'induction', while inventing 'ingoosative' to contrast 'duc(k)' with 'goose', the victim of the force-feeding that leads to the production of $p\hat{a}t\hat{e}$ de foie gras. De Morgan is also hinting at the Wooden Spoon, the title given to the undergraduate who scored the lowest marks in mathematical honours. There is a lot going on in this passage, a nice example of De Morganian deep fun. 40

While most of De Morgan's critical articles on the teaching of mathematics were devoted to Cambridge, two dealt with the Ecole Polytechnique, founded in Paris in 1794, whose institutional and intellectual style offered an interesting contrast to British universities and academies. ⁴¹ Both of these articles were published in the *Quarterly Journal of*

³⁷ De Morgan, 'Wood's Algebra', Quarterly Journal of Education, 3 (1832), 276–85 (p. 276).

³⁸ In his evidence to the 1850 Royal Commission on Cambridge, Whewell declared that 'a permanent board of examiners would be more consistent from year to year, and less affected by peculiar views and habits of the Examiners' (Report of Her Majesty's Commissioners Appointed to Inquire into the State, Discipline, Studies, and Revenues of the University and Colleges of Cambridge: Together with the Evidence, and an Appendix (London: HMSO, 1852), vol. 2, p. 272).

³⁹ See C.A. Stray, 'Rank (dis)order in Cambridge 1753–1909: The Wooden Spoon', *History of Universities*, 26 (2012), 163–201.

⁴⁰ Cambridge, Trinity College Library, Add. MS a.202/114: De Morgan to Whewell, 20 April 1849. The reference is perhaps to Whewell's On the History of the Inductive Sciences or Philosophy of the Inductive Sciences, new editions of which were published in 1847; Whewell's surviving letters to De Morgan do not include any from 1849.

⁴¹ In both institutions, undergraduate slang was permeated by mathematics—for example, the 'argot de l'X' of the Ecole included a formula for the curve of

Education (1831-35), edited by De Morgan's London University colleague, the professor of Greek George Long; De Morgan contributed over thirty articles to the *Journal*.⁴² The first article provided a historical account of the Ecole; the second compared the level and nature of its teaching with that of Cambridge and with that of the Royal Military Academy, Woolwich.⁴³ The Ecole, he concluded, stood somewhere between the two: as an institution founded to train military engineers, it resembled Woolwich; as the home of high-level mathematical analysis, it rivalled Cambridge. De Morgan ended by pointing to a shared weakness:

It is the besetting sin of *all* public places of education to become hotbeds for forcing the first order of talent to the neglect of the rest. ... It must be remembered that those who are pointed out as proofs of a good system, are generally those who would have instructed themselves under any system. The question always should be asked, How are those taught who most want teaching?⁴⁴

This concern for the bulk of ordinary students can also be seen in one of De Morgan's earliest contributions to the *Quarterly Journal of Education*, which ended by identifying

the fault of our schools in general. It is not recollected that they cannot expect to make learned men; but they may make good learners, and at the same time produce such a desire for knowledge as shall lead the individual to devote himself to study, where it is not matter of compulsion, as in the Universities, and still more amid the occupations of life.⁴⁵

the students' uniform cap. On the Ecole and mathematics, see Ivor Grattan-Guinness, *Convolutions in French Mathematics*, 1800–1840, 3 vols. (Basel: Birkhauser Verlag, 1990).

⁴² His contributions are listed in Stray, 'Introduction', *The Quarterly Journal of Education*, 10 vols. (London: Routledge, 2008), vol. 1, pp. xiii–xiv. The founders of the University were also responsible for setting up the Society for the Diffusion of Useful Knowledge, to whose *Penny Cyclopaedia* (1833–43) De Morgan contributed over 700 articles: see Chapters 4 and 8 in this volume.

⁴³ De Morgan, 'Polytechnic School of Paris', Quarterly Journal of Education, 1 (1831), 57–74; 'Ecole Polytechnique', Quarterly Journal of Education, 10 (1835), 330–40.

⁴⁴ De Morgan, 'Ecole Polytechnique', pp. 339–40.

⁴⁵ De Morgan, 'On Mathematical Instruction', *Quarterly Journal of Education*, 1 (1831), 264–79 (p. 279).

One of De Morgan's most eloquent attacks on the Cambridge system came in the form of a mock examination paper, a genre popular among Cambridge undergraduates:

- Q. What is knowledge?
- A. A thing to be examined in.
- Q. What is the instrument of knowledge?
- A. A good grinding tutor.
- Q. What is the end of knowledge?
- A. A place in the civil service, the army, the navy, & (as the case may be).
- Q. What must those do who would show knowledge?
- A. Get up subjects and write them out.
- Q. What is getting up a subject?
- A. Learning to write it out.
- Q. What is writing out a subject?
- A. Showing that you have got it up.46

Although much of De Morgan's criticism was directed at Cambridge, he was also unsparing in his assessment of University College London, as it became in 1836, and of the examining University of London to which it belonged from that year.⁴⁷ The denunciation of cramming quoted above

⁴⁶ S.E. De Morgan, Memoir, p. 184. Sophia De Morgan called this 'an illustrative "Cambridge examination", presumably quoting the author's own title (p. 183). A 'grinder' was a coach or private tutor (see C.A. Stray, Grinders and Grammars: A Victorian Controversy (Reading: Textbook Colloquium, 1995)). An earlier example of the mock-examination paper genre was dated 5 Dec. 1815 but carried the imaginary date 'Undecember 9657'. The questions included: (10) Prove all the roots of radical reform to be either irrational or impossible; (13) Reconcile Hoyle and Euclid, the latter of whom defines a point to be without magnitude, the former to equal five. (Cambridge University Library, Cambridge Papers, MP [unpaginated].) Edmond Hoyle's A Short Treatise on the Game of Whist (1742) was the standard authority on the rules of card games and was not superseded until 1864.

⁴⁷ See in general Rice, 'Augustus De Morgan', pp. 167–73.

from his 1848 introductory lecture surely drew its ammunition from his own experiences in London.

Oxford teaching and examining was very different from that in Cambridge, mathematics being a minority subject, as Classics was in Cambridge. In 1832 De Morgan took the opportunity afforded by reviewing a reformist pamphlet by Baden Powell, Savilian Professor of Geometry, to give a wide-ranging critique of Oxford mathematics. His complaint was less with the teaching than with the dismissive attitude of most Oxford dons toward the subject. In some ways, indeed, he thought the Oxford mode of examination to be superior to those in Cambridge and London. In 1853 De Morgan was asked to specify his objections to the London examination system. In his reply, De Morgan put this in a wider context:

There are two systems in this country,—that of Oxford, in which the candidate for classical honours is examined against his subject; that of Cambridge, in which the candidate for mathematical honours is examined against his competitors. At Oxford, his class determines his qualification; at Cambridge, his place determines whether he is above or below any given competitor. At Oxford his mind may, though not without certain wholesome restraint, develop itself in reading and thought dictated by its natural bent. At Cambridge the examination realised the bed of Procrustes. The Oxford system has a tendency to develop the useful differences between the varied types of human character. The Cambridge system is an unconscious effort to destroy them. ⁵¹

⁴⁸ Keith C. Hannabuss, 'Mathematics', in The History of the University of Oxford VII: Nineteenth-century Oxford, Part 2, ed. by M.G. Brock and M.C. Curthoys (Oxford: Oxford University Press, 2000), pp. 443–55; Christopher Stray, 'Curriculum and Style in the Collegiate University: Classics in Nineteenth-Century Oxbridge', History of Universities, 16 (2001), 183–218; repr in Christopher Stray, Classics in Britain: Scholarship, Education, and Publishing, 1800-2000 (Oxford: Oxford University Press, 2018), pp. 31–52.

⁴⁹ De Morgan, 'State of Mathematical and Physical Sciences in Oxford', *Quarterly Journal of Education*, 4 (1832), 191–208.

⁵⁰ Sophia (*Memoir*, p. 222) identified the recipient of De Morgan's reply as 'Professor Michael Foster', but this cannot be the celebrated physiologist, who was born in 1836 and became a professor only in 1869, nor can it be De Morgan's colleague G.C. Foster, whose chair was awarded in 1865, unless the letter is misdated. She may have confused Foster with his father, also Michael Foster (1810–80), who had been a student at the London University soon after De Morgan was appointed.

⁵¹ De Morgan to Foster, 15 Nov. 1853 (S.E. De Morgan, Memoir, pp. 225–26).

Taking a wider view, De Morgan told William Whewell in 1861 that 'There seems to be a complete acquiescence in the maxim that Oxford shall settle what the world shall think, and Cambridge shall settle who is to be Senior Wrangler. It is getting worse and worse from day to day'.⁵²

This survey of De Morgan's writing on mathematics education began by invoking the remark he made in his first lecture as professor of mathematics at what was then the London University in 1828, that he was determined to teach his students better than his own teachers had taught him. The principle he then enunciated can be applied to his own career, for he surely had opportunity to improve not only on his past teachers, but on his own past self, during his long career in London.⁵³

When De Morgan began teaching in London in 1828, he faced both challenges and opportunities. The challenges came from the uncertainty about his future students' mathematical knowledge, and about his own capacity, as a novice, to teach them. The opportunities arose from the same source: he was a beginner, but in charge of his course, so could make his own curriculum. He began by setting this out, with a list of topics for each of the two years, the junior (first year) and senior (second year) classes each being divided into two semesters. Having found that his students had a wide ability range, he soon split each class into two divisions; he will have remembered from his Cambridge days that college lectures suffered from being mixed-ability classes, in which progress was made at the speed of the slowest. In his first year, then, De Morgan was very much feeling his way; but in the second, he had the advantage of previous experience, and of knowing more about those who were now in his senior class. The London University attached great importance to the utility of examinations, and De Morgan spent a lot of his lecture time on questioning his classes. This oral examination was then supplemented by written tests, which became a marked feature of the University's procedure. The beginning of De Morgan's career came

⁵² De Morgan to Whewell, 20 Jan. 1861 (S.E. De Morgan, *Memoir*, pp. 305–06).

⁵³ On his appointment, see the detailed account in Rice, 'Inspiration or Desperation?'. For more on De Morgan's teaching career, see Adrian Rice, 'What Makes a Great Mathematics Teacher? The Case of Augustus De Morgan', American Mathematical Monthly, 106 (1999), 534–52.

at the point when both Oxford and Cambridge moved decisively away from oral examinations to written tests using printed papers.⁵⁴

De Morgan's return in 1836 to University College London, as it had become, prompted him to give a second inaugural lecture.⁵⁵ In this he expressed his support for some aspects of the Cambridge system of which he had been, in many respects, a critic. In particular, he spoke in favour of the Cambridge study in depth of a few subjects, rather than the wide curricular range proposed for the new University of London, to which University College belonged. On the other hand, he approved of the plan to make the new University an examining institution which left the business of teaching to its constituent colleges (University College and King's College). In this respect, it followed the practice of Cambridge.

Much more evidence is available for De Morgan's second stint as professor of mathematics than for the first. This is because 320 exercise books survive in which he made notes for his lectures. He made them available for students to read, and was thus perhaps harking back to his own Cambridge experience in the 1820s, when student reading centred on the perusal of a mass of manuscript texts written both by college tutors and by private tutors (coaches). In these notebooks we can see De Morgan's exposition at work, as he strove to provide material for students to study at home, one of the two kinds of study he approved of; in 1848 he recommended 'diligent study in the retirement of the closet', along with 'haunting the benches of the lecture-room, and picking up what may chance to fall'. It was in the lecture-room that they were confronted by the tall, stout form and domed forehead of the man known to several

⁵⁴ This involved shifting from the belief that fairness consisted in tailoring questioning to individual candidates' abilities, as could be done in oral questioning, to the belief we now take for granted, that fairness could only be achieved by asking all candidates the same question. For more on this, and on the London University's procedures, see Stray, 'From Oral to Written Examination'.

⁵⁵ De Morgan, Thoughts Suggested by the Establishment of the University of London: An Introductory Lecture, Delivered at the Opening of the Faculty of Arts, in University College, Oct 16, 1837 (London: Taylor & Walton, 1837). Cf. Rice, 'Augustus De Morgan', pp. 155–62.

⁵⁶ Senate House Library, University of London, MS 775. The books are of about 20pp each; the contents have been very well summarised and discussed by Adrian Rice in his 'Augustus De Morgan', pp. 174–203.

⁵⁷ See Wright, *Alma Mater*, vol. 2, p. 24; Warwick, *Masters of Theory*, pp. 71, 144.

⁵⁸ De Morgan, 'On the Effects of Competitory Examinations', p. 14.

generations of students as 'Gussy', the nickname itself an indication of their affection.⁵⁹ His student Stanley Jevons recalled that:

As a teacher of mathematics De Morgan was unrivalled. He gave instruction in the form of continuous lectures delivered extempore from brief notes. The most prolonged mathematical reasoning, and the most intricate formulae, were given with almost infallible accuracy from the resources of his extraordinary memory. De Morgan's writings, however excellent, give little idea of the perspicuity and elegance of his viva voce expositions, which never failed to fix the attention of all who were worthy of hearing him.⁶⁰

Sedley Taylor, who heard De Morgan in the early 1850s, wrote in similar terms:

De Morgan's exposition combined excellences of the most varied kinds. It was clear, vivid, and succinct—rich too with abundance of illustration always at the command of enormously wide reading and an astonishingly retentive memory. A voice of sonorous sweetness, a grand forehead, and a profile of classic beauty, intensified the impression of commanding power which an almost equally complete mastery over Mathematical truth, and over the forms of language in which he so attractively arrayed it, could not fail to make upon his auditors.⁶¹

Taylor's comment on De Morgan's voice is worth noting: the two men shared a love of music (and membership of the Cambridge University Musical Society). Taylor himself published on acoustics, his analysis based on the pioneering work of Hermann von Helmholtz.⁶²

⁵⁹ In his memoir of Cambridge life, J.M.F. Wright remembered that his sympathetic mathematics lecturer John Brown was affectionately known as 'Johnny', whereas for the formal and pompous classical lecturer, James Monk, 'we never used any darling diminutives' (*Alma Mater*, vol. 1, p. 123).

⁶⁰ W.S. Jevons, 'De Morgan, Augustus', *Encyclopaedia Britannica*, 11th edn (Cambridge: Cambridge University Press, 1910), pp. 8–10 (p. 8).

⁶¹ Sedley Taylor, 'Augustus De Morgan', *Cambridge University Reporter*, 3 May 1871, pp. 337–38 (p. 337). Taylor was never formally a pupil of De Morgan's, but attended University College School before matriculating at Cambridge in 1855.

⁶² Sedley Taylor, Sound and Music: An Elementary Treatise on the Physical Constitution of Musical Sounds and Harmony, Including the Chief Acoustical Discoveries of Professor Helmholtz (London: Macmillan, 1873).

Conclusion

De Morgan's students' memories of his teaching resonate with the impression one gains from his letters and his published work.⁶³ To read De Morgan's critiques of mathematical education in Oxford, Cambridge, London and Paris is to be impressed by the sharpness of his perceptions and the down-to-earth clarity with which he expresses them. It makes one regret that he never published his very last introductory lecture at University College, delivered in 1862:

Seldom was an address listened to within those walls with a more lively interest, or with such mirth and hearty acclamation ... The illustrations which half filled the lecture were taken from common sayings, old ballads, and nursery rhymes.⁶⁴

De Morgan was asked to print this lecture, but never found time to do so. As so often, his subject was the method of examining at Cambridge; and as we have seen, this was the major target of his critiques for three decades. As Sophia De Morgan remarked, this was a branch of his favourite subject, education, which we might see as the topic that linked his interests in mathematics and its history: a history which he helped to make.

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⁶³ For students' memories, see Rice, 'Augustus De Morgan', pp. 239–52.

⁶⁴ Memoir, p. 278. Some of De Morgan's notes for this survive: UCL Special Collections, MS Add. 2.

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