

AUGUSTUS
DE MORGAN,
POLYMATH



EDITED BY KAREN ATTAR, ADRIAN RICE
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Selection from 560
English Anagrams on
'Augustus De Morgan'
sent by J. T. Graves Decr. 1863.

A. De Morgan Esquire, M. A.
Professor
= Found queer poems or anagrams

Gus! Gus! a mature don.
August man! sure, god!
Great gun, do us a sum
A dunc rogue at sum
A great go sum $\int \frac{du}{u^n}$
Gus! do an argute sum
Go! great sum $\int a^x du$
Gus! Gus! read amount.
Gus! Gus! read out, man!
O Gus! try a surd. Amen
O Gus! try a mean surd
O! try us a false surd
And Gus must rogue, O!
Must argue on perudo
Gus! to sum and argue
O Gus! Gus! mad nature
Gus! Gus! a true monad.

Fig. 10 Part of a list of 560 English anagrams of De Morgan's name compiled by his friend, fellow mathematician and bibliophile John Thomas Graves in December 1863. Graves created over two thousand permutations in several languages, of which around twenty are featured in *A Budget of Paradoxes*. (MS ADD 7, reproduced by permission of UCL Library Services, Special Collections.)

7. De Morgan's *A Budget of Paradoxes*

Adrian Rice

Great gun, do us a sum!

— One of several anagrams of
'Augustus De Morgan' in *A Budget of Paradoxes*

Introduction

Even the briefest study of Augustus De Morgan's work shows him to have been an intriguing character whose enormous intellect was matched by a sharp wit and keen sense of humour. Although he is best known today for his eponymous laws, for a generation or so after his death one of his main claims to fame was work of a very different character. This was a 500-page book entitled *A Budget of Paradoxes*, published posthumously in 1872, one year after his death, which remains his most accessible and often-quoted work.¹ *A Budget of Paradoxes* arose from De Morgan's journalistic contributions to the weekly London-based literary magazine, *The Athenæum* (an antecedent of today's *New Statesman*), which supplied Victorian Britain with news of the latest developments in the arts, science, and politics.

Over a period of thirty years between 1840 and 1869, De Morgan wrote about one thousand book reviews for *The Athenæum* on a wide variety of topics, from mathematics to history, music and literature.² His

1 The book was originally published as: Augustus De Morgan, *A Budget of Paradoxes* (London: Longmans, Green, 1872). A two-volume second edition from Open Court Publishing, edited and annotated by David Eugene Smith, appeared in 1915. Unless stated otherwise, all citations from the *Budget* will be from the original 1872 edition.

2 Sloan Evans Despeaux and Adrian C. Rice, 'Augustus De Morgan's Anonymous Reviews for *The Athenæum*: A Mirror of a Victorian Mathematician', *Historia*

wide-ranging areas of expertise went hand-in-hand with his standing as one of nineteenth-century Britain's most knowledgeable bibliophiles.³ By the end of his career, De Morgan had accumulated a library that stood at nearly four thousand items, many of which were acquired as complimentary copies of works he had reviewed for *The Athenæum*.

In August 1863, in a letter on the subject of *The Athenæum* to Henry Brougham, a former Lord Chancellor and one of the founders of University College London, De Morgan wrote:

I am on the point of giving, in that paper, a series headed "A Budget of Paradoxes" giving a list, with comments, of all the circle-squarers, universe-builders, &c who are in my library. I think I shall have about 200, including all the rational *paradoxers*, as I call them, who are not much known ... They are a rare lot.⁴

By 'budget', De Morgan meant simply a collection or assortment, whereas the word 'paradox' was used in its archaic sense of 'something which is apart from general opinion, either in subject-matter, method, or conclusion'.⁵ So, for example, theories such as heliocentrism or Darwinism would originally have been classed as paradoxes, because they did not then belong to the mainstream of accepted scientific thought. But although works by mainstream authors do appear in the *Budget*, the focus of De Morgan's attention throughout is the published output of mathematical cranks, frauds and pseudoscientists.⁶ He wrote concerning selection:

To this my answer is, that no selection at all has been made. The books are, without exception, those which I have in my own

Mathematica, 43 (2016), 148–71.

3 See Chapter 10 in this volume.

4 University College London (UCL), Brougham Correspondence, no. 10,307: Letter from Augustus De Morgan to Lord Brougham, 13 Aug. 1863.

5 Augustus De Morgan, *A Budget of Paradoxes* (London: Longmans, Green, 1872), p. 2.

6 While all of the books reviewed in the *Budget* were unorthodox, a few were significant enough to have merited learned discussion in De Morgan's more serious publications. For example, his 1848 paper 'An Account of the Speculations of Thomas Wright of Durham' (*Philosophical Magazine*, 3rd ser., 32 (1848), 241–52) concerned Wright's *Original Theory or New Hypothesis of the Universe* (1750), which had proposed novel ideas concerning nebulae and the Milky Way, later adopted by William Herschel and subsequently by the mainstream astronomical community. De Morgan's entry on Wright's book in the *Budget* was far more cursory (*Budget*, p. 90).

library; and I have taken *all*—I mean all of the kind: Heaven forbid that I should be supposed to have no other books!⁷

De Morgan's *Budget of Paradoxes* column first appeared in *The Athenæum* on Saturday 10 October 1863, running for about three and a half years until 30 March 1867. De Morgan clearly relished his new career as a columnist, since it provided the perfect forum for his whimsical, humorous, eccentric, but above all, entertaining style of writing. As his widow, Sophia, informs us in her preface to the eventual published book: 'the Budget was in some degree a receptacle for the author's thoughts on any literary, scientific, or social question'.⁸ It also proved provocative, since several of its readers were compelled to write to *The Athenæum*, either to protest at their inclusion or to demand insertion in a subsequent issue.

After the publication of the concluding column in 1867, De Morgan continued to collect material and write further additions to the *Budget*, which he inserted piecemeal into what grew to be a lengthy and complex manuscript. It was this manuscript which served as the basis of the book that appeared in 1872 'reprinted, with the author's additions, from the "Athenæum".' Although noting apologetically that De Morgan's work on the book's preparation was incomplete at the time of his death, its editor, Sophia De Morgan, expressed the hope that 'it will be welcomed as an old friend returning under great disadvantages, but bringing a pleasant remembrance of the amusement which its weekly appearance in the *Athenæum* gave to both writer and reader'.⁹

That hope was certainly achieved. Subsequent reviews were glowing in their praise and the fondness with which the book was regarded was near universal. The mathematician William Clifford described the book as 'by very far the most individual book of the age', while P. G. Tait called it 'absolutely unique'.¹⁰ Indeed the Scottish mathematician Alexander Macfarlane later recalled that when he was a student at the University of Edinburgh, Tait once said to him: 'If you wish to read something

7 *Budget*, pp. 5–6.

8 Sophia Elizabeth De Morgan, 'Editor's Preface', in *Budget*, pp. v–vii (p. vi).

9 *Budget*, p. vii.

10 [William Kingdon Clifford], Review of *A Budget of Paradoxes*, *The Academy* (15 Aug. 1873), 306–07 (p. 306) [repr. in *Mathematical Papers* (London: Macmillan, 1882), pp. 559–61 (p. 559)]; Peter Guthrie Tait, 'De Morgan's *Budget of Paradoxes*', *Nature* (30 Jan. 1873), 239–40 (p. 239).

entertaining, get De Morgan's *Budget of Paradoxes* out of the library.¹¹ What then was the attraction of *A Budget of Paradoxes*, and what made it so unusual? To answer these questions, let us take a brief look at what David Eugene Smith described as 'one of the most delicious bits of satire of the nineteenth century'.¹²

A Brief Survey of the *Budget*

De Morgan opens his *Budget of Paradoxes* with the following idiosyncratic words:

If I had before me a fly and an elephant, having never seen more than one such magnitude of either kind; and if the fly were to endeavour to persuade me that he was larger than the elephant, I might by possibility be placed in a difficulty. The apparently little creature might use such arguments about the effect of distance, and might appeal to such laws of sight and hearing as I, if unlearned in those things, might be unable wholly to reject. But if there were a thousand flies, all buzzing, to appearance, about the great creature; and, to a fly, declaring, each one for himself, that he was bigger than the quadruped; and all giving different and frequently contradictory reasons; and each one despising and opposing the reasons of the others—I should feel quite at my ease. I should certainly say, My little friends, the case of each one of you is destroyed by the rest. I intend to show flies in the swarm, with a few larger animals, for reasons to be given.¹³

The book that follows is essentially an ordered list of reviews of 270 publications presented roughly in chronological order, with the earliest item being an incunabulum from 1489 and the latest a work dating from 1866. The reviews vary in length from one or two lines to several pages and are interspersed here and there with tangential remarks, excerpts from articles, witty verse, puzzles, puns, amusing anecdotes, and very many digressions. These include discussions on probability and the

11 Alexander Macfarlane, *Lectures on Ten British Mathematicians of the Nineteenth Century* (New York: Wiley, 1916), p. 25.

12 David Eugene Smith, 'Preface to the Second Edition', in Augustus De Morgan, *A Budget of Paradoxes*, 2nd edn (Chicago: Open Court Publishing, 1915), pp. vi–viii (p. viii).

13 *Budget*, p. 1.

law of large numbers, the Buffon needle problem, Wilson's theorem in number theory, the controversy over the discovery of Neptune, the merits of a proposed system of decimal coinage, the veracity of the story of Newton and the apple, the etymology of various words, and several metaphysical considerations. For example, in a review of a philosophical work, *The Mystery of Being; Or are Ultimate Atoms Inhabited Worlds?*, De Morgan parodies a verse by Jonathan Swift:

Great fleas have little fleas, upon their backs to bite 'em,
And little fleas have lesser fleas, and so *ad infinitum*.¹⁴

A multitude of rogue theories are presented from a host of academic disciplines including mathematics, physics, astronomy, medicine, economics, logic, philosophy, and theology. Such theories include attempts to trisect the angle, to construct a perpetual motion machine, to prove that the earth is flat, to disprove the law of universal gravitation and, of course, to square the circle—and all are beautifully sent up by De Morgan in his wonderfully satirical style. *Comets Considered as Volcanoes, and the Cause of their Velocity and Other Phenomena Thereby Explained* receives the review: 'The title explains the book better than the book explains the title';¹⁵ while a reference to an alleged eight-volume work on parallel lines produces the remark: 'Surely this is a misprint; eight volumes on the theory of parallels? If there be such a work, I trust I and it may never meet, though ever so far produced.'¹⁶ Perhaps the most poignant remarks concern a pamphlet by a Mr. James Sabben, *A Method to Trisect a Series of Angles Having Relation to Each Other; Also Another to Trisect Any Given Angle* (1848), described by its author as 'The consequence of years of intense thought'. De Morgan's review, in its entirety, reads: 'Very likely, and very sad.'¹⁷

One of the most famous works reviewed is Robert Chambers' *Vestiges of the Natural History of Creation*. Published anonymously in

14 *Budget*, p. 377. Swift's original verse in *On Poetry: A Rapsody* (1733) was:
So, Nat'ralists observe, a Flea
Hath smaller Fleas that on him prey,
And these have smaller Fleas to bite 'em,
And so proceed *ad infinitum*.

15 *Budget*, p. 303.

16 *Budget*, p. 137.

17 *Budget*, p. 255.

1844, this controversial book began the popularisation of the idea of the transmutation of species and helped pave the way, in the public eye at least, for the subsequent acceptance of Darwin's theories of evolution by natural selection. Unsurprisingly, speculation about the book's authorship was rife for many years, with the truth only revealed in 1884, thirteen years after the deaths of both Chambers and De Morgan.¹⁸ In his review, De Morgan confessed:

I never hear a man of note talk fluently about it without a curious glance at his proportions, to see whether there may be ground to conjecture that he may have more of 'mortal coil' than others ... [w]ith a hole behind which his tail peeped through.¹⁹

His only criticism of the book concerned an inaccuracy in its very first sentence, which read: 'It is familiar knowledge that the earth which we inhabit is a globe of somewhat less than 8,000 miles in diameter, being one of a series of eleven which revolve at different distances around the sun.'²⁰ 'The *eleven!*' De Morgan exclaimed, 'Not to mention the Iscariot which Le Verrier and Adams calculated into existence, there is more than a septuagint of *new* planetoids.'²¹

He expressed similar sentiments in a review of *The Decimal System as a Whole* (1856), a book written in support of the adoption of the decimal system, but which perhaps took the idea a little too far:

18 James A. Secord, *Victorian Sensation: The Extraordinary Publication, Reception, and Secret Authorship of Vestiges of the Natural History of Creation* (Chicago: University of Chicago Press, 2003).

19 *Budget*, p. 211.

20 [Robert Chambers], *Vestiges of the Natural History of Creation* (London: John Churchill, 1844), p. 1.

21 *Budget*, p. 211. There is a lot going on in this sentence. The eleven celestial bodies referred to by the author of *Vestiges* are seven planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn and Uranus) and the four then-known asteroids (Ceres, Pallas, Juno and Vesta). The planet Neptune, discovered in 1846 via the mathematical calculations of Urbain Le Verrier and John Couch Adams, constituted the twelfth known body to be orbiting the sun, hence De Morgan's reference to the twelfth disciple, Judas Iscariot. A further biblical allusion occurs in his use of the word 'septuagint'. Although usually understood, when capitalised, to mean the Greek translation of the Old Testament, the word literally means seventy. By the time De Morgan was writing, in the mid-1860s, astronomers had discovered around 100 asteroids in the solar system. He would thus have been justified in viewing *Vestiges'* enumeration of eleven planetary bodies as somewhat out of date.

The proposition is to make everything decimal. The day, now 24 hours, is to be made 10 hours. The year is to have ten months, Unusber, Duoher, &c. Fortunately there are ten commandments, so there will be neither addition to, nor deduction from, the moral law. But the twelve apostles! Even rejecting Judas, there is a whole apostle of difficulty. These points the author does not touch.²²

Several paradoxes focus on attempts to apply mathematics, not always successfully, to areas hitherto untouched by the subject. One such area was psychology, on which the German philosopher and psychologist Johann Friedrich Herbart published *De Attentionis Mensura Causisque Primariis* in 1822. In it Herbart laboured to derive mathematical formulae for various attributes of concentration. His success was marred only by their unintelligibility, as De Morgan writes:

As a specimen of his formula, let t be the time elapsed since the consideration began, β the whole perceptive intensity of the individual, ϕ the whole of his mental force, and z the force given to a notion by attention during the time t . Then,

$$z = \phi(1 - e^{-\beta t})$$

Now for a test. There is a *jactura*, v , the meaning of which I do not comprehend.²³ If there be anything in it, my mathematical readers ought to interpret it from the formula

$$v = \frac{\pi\phi\beta}{1 - \beta} e^{-\beta t} + C e^{-t}$$

and to this task I leave them, wishing them better luck than mine.²⁴

Theology was another subject which appeared ripe for mathematicising, as evinced by such works as John Craig's *Theologiae Christianiae Principia Mathematica* (1699) and Richard Jack's *Mathematical Principles of Theology* (1747). Not only were these titles in direct emulation of Newton's *Philosophiae Naturalis Principia Mathematica*, but their content was clearly influenced by the then novel idea that concepts such as force were not only measurable but subject to mathematical laws of quantifiable

²² *Budget*, p. 301.

²³ As a scholar well versed in Classics, De Morgan would have understood *jactura* to be a Latin word meaning loss or expense; his confusion presumably arose from the question of how this term related to the subject of concentration.

²⁴ *Budget*, p. 150.

variation. In his book, Craig, himself an able mathematician, took as an axiom that the rate at which suspicions against historical evidence increase is proportional to the square of the time elapsed. On this hypothesis, he attempted to calculate how long it would take the evidence of Christianity to die out, coming up with a date of AD 3150, which he also gave as the year of the second coming. 'It is a pity that Craig's theory was not adopted,' De Morgan commented drily, 'it would have spared a hundred treatises on the end of the world'.²⁵

But while some authors wished to emulate the work of Isaac Newton, there were plenty in the eighteenth and nineteenth centuries who wished to prove him wrong. The Italian Caelestino Cominale published his *Antinewtonianismus* in two hefty volumes in 1754 and 1756, in which he sought to refute the Newtonian theories of light, inertia, vacua and gravitation. Although they occupied a place on De Morgan's shelves, he had to admit: 'I never attempted these big Latin volumes, numbering 450 closely-printed quarto pages. The man who slays Newton in a pamphlet is the man for me.'²⁶ Such a man was one Captain Walter Forman, a retired Royal Naval officer from Shepton Mallet, who published an anti-Newtonian pamphlet in 1833, *A Letter to the Secretary of the Royal Astronomical Society, in Refutation of Some Absurd & Mistaken Notions, upon Philosophical Subjects which are Held in Common by that Society and by all the Newtonian philosophers*.

De Morgan clearly found this easier reading than its Latin counterpart, since he tells us that he was 'happy to state that there is no truth in the rumour of the laws of gravitation being about to be repealed. We have traced this report, and find it originated with a gentleman living near Bath (Captain Forman, R.N.), whose name we forbear to mention.'²⁷ A quicker read still was a flyer handed out on the street sometime in 1847 which read:

Important discovery in astronomy, communicated to the Astronomer Royal, December 21st, 1846. That the Sun revolve round the Planets in 25748² years, in consequence of the combined Attraction of the Planets and their Satellites, and that the Earth revolve round the Moon in 18 years and 228 days.²⁸

²⁵ *Budget*, p. 77.

²⁶ *Budget*, p. 96.

²⁷ *Budget*, p. 185.

²⁸ *Budget*, p. 253.

The Morbus Cyclometricus

If subverters of Newtonian physics were a sizeable constituent of De Morgan's paradoxers, by far the most numerous were those amateur mathematicians who claimed to have squared the circle or, to put it in modern terminology, to have found a rational value of π . The *Budget* dealt with more than fifty such works, including two books by J. P. de Fauré, an obscure Swiss mathematician, published in 1747 and 1749. Both claimed to have proved that the true value of π is 256/81. But De Morgan observed, much to his surprise, that the second of the two volumes carried printed endorsements from two far more eminent Swiss mathematicians, namely Johann Bernoulli the younger and Samuel König. However, on closer inspection, he noticed that these endorsements were very cunningly worded. Bernoulli's testimonial reads: 'Under the assumptions framed in this memoir, it [the conclusion] is so obvious that ... it needs neither evidence nor authority to be recognized by everyone',²⁹ while König's is even more evasive: 'I concur with the judgment of Mr. Bernoulli, in consequence of these assumptions.'³⁰ This verbal dexterity prompted De Morgan to write: 'It should seem that it is easier to square the circle than to get round a mathematician.'³¹

But by far the most notorious circle-squarer, as well as the most voluminous contributor to the *Budget of Paradoxes*, other than De Morgan himself, was a certain James Smith. One of the most bizarre, irrepressible and pugnacious figures in the history of mathematics, Smith was in De Morgan's words a man who 'more visibly than almost any other known to history, reasoned in a circle by way of reasoning on a circle'.³² De Morgan was clearly wholly unprepared for the magnitude of Smith's mathematical inabilities:

I had not anything like an adequate idea of Mr. James Smith's superiority to the rest of the world in the points in which he is superior. He is beyond doubt the ablest head at unreasoning, and the greatest hand at writing it, of all who have tried in our day

29 'Suivant les suppositions posées dans ce Mémoire, il est si évident que ... cela n'a besoin ni de preuve ni d'autorité pout être reconnu par tout le monde.'

30 'Je souscris au jugement de M. Bernoulli, en conséquence de ces suppositions.'

31 *Budget*, p. 90.

32 *Budget*, p. 331.

to attach their names to an error. Common cyclometers sink into puny orthodoxy by his side.³³

Smith had been a successful Liverpool merchant who chaired the local marine board for several years as well as being a member of the Mersey Docks and Harbour Board.³⁴ He possessed an avocational interest in mathematics and, following his retirement in 1855, became interested in the problem of squaring the circle. Believing that he had discovered the true value of π to be precisely $3\frac{1}{8}$, in 1859 Smith privately published *The Problem of Squaring the Circle Solved*, which De Morgan reviewed in *The Athenæum* on 5 March of that year. Not surprisingly, his review was hardly a ringing endorsement of Smith's work, although it did state that

we by no means desire to prevent any one who is not deep in mathematics from squaring the circle. It is a mode of meddling with unknown things which cannot do any harm, except to the speculator himself. If Mr. Smith will study geometry, he will find out his own fallacy fast enough.³⁵

Smith did not find his own fallacy and appears to have entered into a private correspondence with De Morgan in which the mathematician attempted (valiantly, though unsuccessfully) to persuade Smith of the errors in his reasoning.³⁶ It quickly became evident that Smith's 'proof' rested on an initial assumption that $\pi = 3\frac{1}{8}$, followed by the deduction of a few consequences consistent with that hypothesis. To a mathematician and logician of De Morgan's calibre, this was too much. He later wrote:

Euclid assumes what he wants to *disprove*, and shows that his *assumption* leads to absurdity, and so *upsets itself*. Mr. Smith assumes what he wants to *prove*, and shows that *his* assumption makes *other propositions* lead to absurdity. This is enough for all who can reason. Mr. James Smith cannot be argued with ...³⁷

³³ *Budget*, p. 317.

³⁴ E. I. Carlyle, 'Smith, James (1805–1872)', rev. by Adrian Rice, *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004), <https://doi.org/10.1093/ref:odnb/25824>.

³⁵ [Augustus De Morgan], Review of *The Problem of Squaring the Circle Solved*, *The Athenæum* (5 Mar. 1859), 319.

³⁶ No letters in this correspondence appear to be extant from either party. However, see contemporaneous letters from Smith to William Hepworth Dixon, editor of *The Athenæum*, arguing against De Morgan's position on the question of squaring the circle (University College London, MS ADD 118).

³⁷ *Budget*, p. 327.

De Morgan cannot, therefore, have been surprised that Smith remained unconvinced by his arguments; but he was taken aback two years later when Smith published De Morgan's private letters to him in *The Quadrature of the Circle: Correspondence between an Eminent Mathematician and J. Smith, Esq.*³⁸ De Morgan responded with an extended review in *The Athenæum*, later reprinted in the *Budget*. Charging Smith with violating 'the decencies of private life' by the unauthorised publication of private correspondence, De Morgan declared that he 'deserves the severest castigation; and he will get it'.³⁹ Yet he could resort to little but ridicule. While mockingly noting that Smith was clearly suffering from a condition he denoted as 'the *morbus cyclometricus*,' (the circle-measuring disease), De Morgan reassured readers of the *Budget* that Smith 'is not mad. Madmen reason rightly upon wrong premises: Mr. Smith reasons wrongly upon no premises at all.'⁴⁰

Smith replied with letters and advertisements published in various journals and newspapers, including *The Athenæum*, where he outlined his argument and claimed 'for your readers the right and the opportunity of judging for themselves, whether they would desire to class me in the category of either fools or madmen'.⁴¹ De Morgan responded by including in the *Budget* a simple proof sent to him by William Rowan Hamilton that the circumference is greater than $3\frac{1}{8}$ diameters, based only on the first four books of Euclid, saying: 'We give it in brief as an exercise for our juvenile readers ...' He continued:

It reminds us of the old days when real geometers used to think it worth while seriously to demolish pretenders. Mr. Smith's fame is now assured: Sir W. R. Hamilton's brief and easy exposure will procure him notice in connexion with this celebrated problem, to the historians of which we now hand him over.⁴²

38 James Smith, *The Quadrature of the Circle: Correspondence between an Eminent Mathematician and J. Smith, Esq* (London: Simpkin, Marshall, 1861). De Morgan's copy encloses press clippings, letters from William Rowan Hamilton to Augustus De Morgan, 24 Apr.–13 May 1861) and copies of letters from Hamilton to the editor of the *Athenæum* (1–6 May 1861). See Senate House Library, University of London, [DeM] L.6 [Quadrature] SSR.

39 *Budget*, p. 319.

40 *Budget*, pp. 318, 319.

41 James Smith, [Advertisement] 'The Quadrature of the Circle. To the Editor of the *Athenæum*', *The Athenæum* (25 May 1861), 679.

42 [Augustus De Morgan], 'The Circle', *The Athenæum* (8 June 1861), 764.

Other mathematical colleagues rallied to support De Morgan when Smith submitted a paper on the subject for presentation at the 1861 meeting of the British Association for the Advancement of Science in Manchester. After realising that Smith's legitimate-sounding 'The Relations of a Circle Inscribed in a Square' contained nothing but nonsensical ramblings about π , the Association rejected it, conscious of the serious harm such a paper could do to the reputation of mathematics at such a prestigious meeting. Indeed, in his presidential speech to the mathematical and physical section of the Association, mathematician and astronomer George Airy made a point of insisting 'that such communications should not be made to [this] Section, as they were a mere loss of time'.⁴³

Despite this curt dismissal, Smith continued to publish books, pamphlets and letters, in which extracts from correspondence with De Morgan, Hamilton and others were interspersed with further fruitless attempts to argue his case. As time passed, occasional bizarre outbursts of frustration punctured these mathematical arguments, such as Smith's peculiar depiction of De Morgan as a 'Mathematical elephant ... pumping your brains ... behind the scenes'.⁴⁴ De Morgan retorted: 'an odd thing for an elephant to do, and an odd place to do it'.⁴⁵

De Morgan continued to wrestle good-naturedly with him for several more years via both the *Budget* and the columns of his *Athenæum* reviews, although towards the end of his life, he did wonder whether Smith's incessant correspondence would cease only on the occasion of his (De Morgan's) death:

And this time may not be far off: for I was X years old in A.D. X^2 ; not 4 in A.D. 16, nor 5 in A.D. 25, but still in one case under that law. And now I have made my own age a problem of quadrature, Mr. J. Smith may solve it. But I protest against his method of assuming a result, and making itself prove itself: he might in this way, as sure as eggs is eggs (a corruption of X is X), make me 1,864 years old, which is a great deal too much.⁴⁶

43 George Biddell Airy, 'Address by G.B. Airy, Astronomer Royal, President of the [Mathematics and Physics] Section', in *Report of the Thirty-First Meeting of the British Association for the Advancement of Science* (London: John Murray, 1862), pp. 1–2 (p. 2).

44 James Smith, *The Quadrature of the Circle* (Liverpool: Edward Howell, 1865), p. 55.

45 [Augustus De Morgan], Review of *The Quadrature of the Circle*, *The Athenæum* (27 May 1865), 717.

46 *Budget*, p. 332.

Astronomers and Alcohol

De Morgan used the *Budget* not only as a forum for the debunking of pseudoscientific nonsense, but as a repository for some of his own discursions on the history of science and mathematics. Thus, among many other things, we find a lengthy discussion on the history and calculation of the date of Easter, a description of Napier's theological work on the book of Revelations, and a charming account of the now-famous Spitalfields Mathematical Society, which existed from 1717 to 1845. Established as a club for the improvement of the studious artisan, especially the silk weavers of East London, the Society's great rule was: 'If any member be asked a question in the Mathematics by another, he shall instruct him in the plainest and easiest method he can, or forfeit one penny'.⁴⁷ Of their weekly meetings in Crispin Street, East London, De Morgan noted 'that each man had his pipe, his pot, and his problem'.⁴⁸

The Society's members included the optician John Dollond, the mathematician Thomas Simpson, and De Morgan's own father-in-law, William Frend. Its final president was Benjamin Gompertz, an actuarial mathematician and friend of De Morgan, who provided some fascinating information on the background to a particular song, apparently sung at one of the Society dinners around 1800, and reproduced in full in *A Budget of Paradoxes*.⁴⁹ Entitled 'The Astronomer's Drinking Song' it was presumably sung to the tune of 'The Vicar of Bray':⁵⁰

Whoe'er would search the starry sky,
Its secrets to divine, sir,
Should take his glass—I mean, should try
A glass or two of wine, sir!
True virtue lies in golden mean,
And man must wet his clay, sir;
Join these two maxims, and 'tis seen
He should drink his bottle a day, sir!

47 J. W. S. Cassels, 'The Spitalfields Mathematical Society', *Bulletin of the London Mathematical Society*, 11 (1979), 241–58 (p. 244). See also Larry Stewart and Paul Weindling, 'Philosophical Threads: Natural Philosophy and Public Experiment among the Weavers of Spitalfields', *British Journal for the History of Science*, 28 (1995), 37–62 (pp. 41–42).

48 *Budget*, p. 232.

49 *Budget*, pp. 234–36.

50 We give an edited version: the full song has eleven verses.

When Ptolemy, now long ago,
 Believed the earth stood still, sir,
 He never would have blundered so,
 Had he but drunk his fill, sir:
 He'd then have felt it circulate,
 And would have learnt to say, sir,
 The true way to investigate
 Is to drink your bottle a day, sir!

Copernicus, that learned wight,
 The glory of his nation,
 With draughts of wine refreshed his sight,
 And saw the earth's rotation;
 Each planet then its orb described,
 The moon got under way, sir;
 These truths from nature he imbibed
 For he drank his bottle a day, sir!

Poor Galileo, forced to rat
 Before the Inquisition,
E pur si muove was the pat
 He gave them in addition:
 He meant, whate'er you think you prove,
 The earth must go its way, sirs;
 Spite of your teeth I'll make it move,
 For I'll drink my bottle a day, sirs!

Great Newton, who was never beat
 Whatever fools may think, sir;
 Though sometimes he forgot to eat,
 He never forgot to drink, sir:
 Descartes took nought but lemonade,
 To conquer him was play, sir;
 The first advance that Newton made
 Was to drink his bottle a day, sir!

D'Alembert, Euler, and Clairaut,
 Though they increased our store, sir,
 Much further had been seen to go
 Had they tiddled a little more, sir!
 Lagrange gets mellow with Laplace,
 And both are wont to say, sir,
 The *philosophe* who's not an ass
 Will drink his bottle a day, sir!

The fact that smoking and drinking were permitted at meetings of the Spitalfields Society contrasted sharply with the more sober gatherings of its eventual successor, the London Mathematical Society. Writing in 1866, De Morgan stated proudly that

There is a new Mathematical Society, and I am, at this present writing, its first President. We are very high in the newest developments, and bid fair to take a place among the scientific establishments. ... But not a drop of liquor is seen at our meetings, except a decanter of water: all our heavy is a fermentation of symbols; and we do not draw it mild.⁵¹ There is no penny fine for reticence or occult science; and as to a song! not the ghost of a chance.⁵²

Conclusion

As a literary work, *A Budget of Paradoxes* is something of a paradox (in the modern sense) itself: a learned volume yet written in a popular style, serious but funny, respectful but subversive. And although time has inevitably dated its content in terms of allusions to the science, literature, and politics of the day, some parts are still extremely amusing. De Morgan clearly had not only a wicked sense of humour, but a very engaging style as well. He obviously enjoyed telling a good story and one feels very much when reading the book as if he was writing exactly as he would have spoken.

Writing in *Nature* in January 1873, P. G. Tait was fulsome in his praise of the *Budget*. 'Nothing,' he opined, 'in the slightest degree approaching it in its wonderful combinations has ever, to our knowledge, been produced.'⁵³ He observed that De Morgan's keen intellect, jocular writing style and even-tempered disposition made him the ideal author of such a book: 'And every page of it shows that he thoroughly enjoyed his task.'⁵⁴ In his mode of presentation, Tait likened De Morgan to a puppeteer, playing 'with his puppets, showing off their peculiarities,

51 In this pun-loaded sentence, 'heavy' is a Scottish term for a type of medium-strength beer known in England as 'bitter', as opposed to 'mild' ales, which are generally lower in alcohol content.

52 *Budget*, p. 236.

53 Tait, 'De Morgan's *Budget of Paradoxes*', p. 239.

54 Tait, p. 239.

posing them, helping them when diffident, restraining them when noisy, and even occasionally presenting himself as one of their number'.⁵⁵ He went on to note the uniformly good-humoured nature displayed in De Morgan's writing, 'so that the only incongruities we are sensible of are the sometimes savage remarks which several of his pet bears make about their dancing master'.⁵⁶

In a review in *The Academy*, published seven months later, William Clifford praised the book for 'helping us to extend the habits of right thinking which we have got by practice in one subject over the whole range of our knowledge', and said it should be read 'by those who care to be led into right thinking and warned from wrong'.⁵⁷ Yet he noted a logical error in De Morgan's reasoning at one point in the book. One work reviewed in the *Budget, From Matter to Spirit*, concerned alleged manifestations of spiritual beings.⁵⁸ Its anonymous author was Sophia De Morgan ('C. D.'), with a preface by her husband ('A. B.') in which, although he displayed a cautious scepticism, he maintained that some of the phenomena could only have been caused by either an 'unseen intelligence' or something as yet unconceived by man.⁵⁹ But as Clifford pointed out: 'This apparently suspended judgment involves and hides the assumption that the said phenomena cannot possibly be referred to certain well known and commonly conceived things—the art of the conjuror, and the delusion of contagious excitement.'⁶⁰

In 1940 Dirk Struik noticed a further error, this time historical.⁶¹ In two separate sections of the *Budget*, De Morgan regales the reader with a humorous anecdote regarding an alleged encounter between the French philosopher Denis Diderot and the Swiss mathematician Leonhard Euler:

The following story is told by Thiébauld, in his *Souvenirs de vingt ans de séjour à Berlin*, published in his old age, about 1804.⁶² ...
Diderot paid a visit to the Russian Court at the invitation of

55 Tait, p. 239.

56 Tait, p. 239.

57 [Clifford], Review of *Budget*, p. 307.

58 For more on this book, see Chapter 9 of this volume.

59 [Sophia Elizabeth De Morgan], *From Matter to Spirit* (London: Longman, Green, Longman, Roberts, & Green, 1863), pp. v–xlv (p.xxvii).

60 Clifford, p. 307.

61 Dirk J. Struik, 'A Story Concerning Euler and Diderot', *Isis*, 31 (1940), 431–32.

62 Dieudonné Thiébauld (1733–1807), a French man of letters who spent many years at the court of Frederick the Great.

the Empress. He conversed very freely, and gave the younger members of the Court circle a good deal of lively atheism. The Empress was much amused, but some of her councillors suggested that it might be desirable to check these expositions of doctrine. The Empress did not like to put a direct muzzle on her guest's tongue, so the following plot was contrived. Diderot was informed that a learned mathematician was in possession of an algebraical demonstration of the existence of God, and would give it him before all the Court, if he desired to hear it. Diderot gladly consented: though the name of the mathematician is not given, it was Euler. He advanced towards Diderot, and said gravely, and in a tone of perfect conviction: *Monsieur,*

$$\frac{a + b^n}{n} = x,$$

donc Dieu existe; répondez! Diderot, to whom algebra was Hebrew, was embarrassed and disconcerted; while peals of laughter rose on all sides. He asked permission to return to France at once, which was granted.⁶³

This story was later repeated faithfully in print in the first half of the twentieth century by several historians of mathematics, including Florian Cajori, David Eugene Smith and E. T. Bell, as well as by the mathematical populariser Lancelot Hogben in his *Mathematics for the Million*.⁶⁴ But it does not make sense. Euler would have known that Diderot was one of the most intelligent people on the continent and that he was familiar, not just with algebra, but with many areas of mathematics, having written among other things on involutes and probability. For his part, Diderot would not have been swayed for one moment by such an unconvincing trick as Euler's equation. But more than this, at no point in Thiébauld's original account is Euler actually mentioned at all—his insertion in the story is purely an invention by De Morgan.⁶⁵ Surprising for a historian who was usually scrupulous about checking his sources, it serves as a rare example of De Morgan not letting the facts get in the way of a good story.

63 *Budget*, pp. 250–51. The story is told again on p. 474.

64 Florian Cajori, *A History of Mathematics*, 2nd ed. (New York: Macmillan, 1919), p. 233; David Eugene Smith, *History of Mathematics* (Boston: Ginn, 1923), vol. 1, pp. 522–23; E. T. Bell, *Men of Mathematics* (New York: Simon & Schuster, 1937), pp. 146–47; Lancelot Hogben, *Mathematics for the Million* (London: Allen & Unwin, 1936), pp. 13–14.

65 Dieudonné Thiébauld, *Mes Souvenirs de vingt ans de séjour à Berlin*, 5 vols. (Paris: Buisson, 1804–05), vol. 3, pp. 140–43.

By Struik's time, of course, knowledge of many of the allusions current at the time of the *Budget's* writing were quickly fading from living memory. For this reason the *Budget* did not age well, and half a century after its first publication in *The Athenæum*, David Eugene Smith was writing: 'Many books that were then current have now passed out of memory, and much that agitated England in De Morgan's prime seems now like ancient history.'⁶⁶ So much of the content is framed in the context of political or cultural references that were soon largely forgotten. How many today would know, for example, that when De Morgan refers to 'the lady in Cadogan Place' he is writing about Mrs. Witterly from *Nicholas Nickleby*?⁶⁷ Or that 'Miss Pickle, in the novel of that name' is actually Mrs. Grizzle from *The Adventures of Peregrine Pickle*, a picaresque tale by Tobias Smollett, first published in 1751?⁶⁸ For reasons such as these, Smith produced, in 1915, a new edition of the *Budget*, in two volumes with numerous additional footnotes containing valuable contextual information and references.

Reviewing this new edition for the American Mathematical Society in 1916, Louis Karpinski wryly observed: 'A modern De Morgan would, in two volumes like these, have room only for titles of published nonsense.'⁶⁹ Continuing the theme in the journal *Science*, Karpinski lamented the torrent of 'paradoxical nonsense, foisted upon the press by authors ignorant of what has been done by others in the fields in which these authors would instruct the public'.⁷⁰ Among the offenders, he listed 'philosophers ignorant of the work of Georg Cantor and Dedekind who wish to instruct mathematicians about the nature of the number idea and the psychology of number, school superintendents who are profoundly ignorant of the fundamental ideas of arithmetic who wish to write text-books on arithmetic, old maids living in a two-room flat

66 Smith, 'Preface to the Second Edition', p. vii.

67 *Budget*, p. 4. The fact that De Morgan added a footnote in the 1860s to clarify this phrase suggests that he considered the allusion too obscure for contemporary readers to figure out.

68 *Budget*, p. 89. De Morgan provided no footnote or explanatory passage for this reference, suggesting that the book was considered well enough known for the allusion to stand unexplained.

69 [Louis C. Karpinski], Review of *A Budget of Paradoxes*, *Bulletin of the American Mathematical Society*, 22 (1916), 468–71 (p. 471).

70 [Louis C. Karpinski], Review of *A Budget of Paradoxes*, *Science*, 42 (19 Nov. 1915), 729–31 (p. 730).

on the fifteenth floor of a New York apartment who wish to instruct the parents of the United States on the art of bringing up a large family of children ...' and the scores of delusional 'mathematicians' who claimed to have proved Fermat's Last Theorem.⁷¹

Today, the situation is little different. With his long view of history, De Morgan might say 'Twas ever thus.' And with a multitude of contemporary unorthodox theories like climate-change denial, creationism and scientology, it is clear that the phenomenon of the paradoxer has not died down with time. Indeed, with real science often indistinguishable in the public mind from pseudoscience, it is arguable that the need for a modern-day De Morgan to set the record straight has never been greater.

Bibliography

- Airy, George Biddell, 'Address by G.B. Airy, Astronomer Royal, President of the [Mathematics and Physics] Section', in *Report of the Thirty-First Meeting of the British Association for the Advancement of Science* (London: John Murray, 1862), pp. 1–2.
- Bell, Eric Temple, *Men of Mathematics* (New York: Simon & Schuster, 1937).
- Cajori, Florian, *A History of Mathematics*, 2nd edn (New York: Macmillan, 1919).
- Carlyle, E. I., 'Smith, James (1805–1872)', rev. by Adrian Rice, *Oxford Dictionary of National Biography* (Oxford: Oxford University Press, 2004), vol. 51, p. 189. <https://doi.org/10.1093/ref:odnb/25824>
- Cassels, J. W. S., 'The Spitalfields Mathematical Society', *Bulletin of the London Mathematical Society*, 11 (1979), 241–58.
- [Chambers, Robert], *Vestiges of the Natural History of Creation* (London: John Churchill, 1844).
- Clifford, William Kingdon, Review of *A Budget of Paradoxes*, *The Academy* (15 Aug. 1873), 306–07 [repr. in *Mathematical Papers* (London: Macmillan, 1882), pp. 559–61].
- De Morgan, Augustus, *A Budget of Paradoxes* (London: Longmans, Green, 1872). 2nd edn, ed. by David Eugene Smith, 2 vols. (Chicago: Open Court Publishing, 1915).
- , Review of *The Problem of Squaring the Circle solved*, *The Athenæum* (5 Mar. 1859), 319.

⁷¹ [Karpinski], Review, *Science*, p. 730.

- , 'The Circle', *The Athenæum* (8 June 1861), 764.
- , Review of *The Quadrature of the Circle*, *The Athenæum* (27 May 1865), 717.
- [De Morgan, Sophia Elizabeth], *From Matter to Spirit. The Result of Ten Years' Experience in Spirit Manifestations. Intended as a Guide to Enquirers* (London: Longman, Green, Longman, Roberts & Green, 1863).
- Despeaux, Sloan Evans, and Adrian C. Rice, 'Augustus De Morgan's Anonymous Reviews for *The Athenæum*: A Mirror of a Victorian Mathematician', *Historia Mathematica*, 43 (2016), 148–71. <https://doi.org/10.1016/j.hm.2015.09.001>
- Hogben, Lancelot, *Mathematics for the Million* (London: George Allen & Unwin, 1936).
- Karpinski, Louis C., Review of *A Budget of Paradoxes*, *Science*, 42 (19 Nov. 1915), 729–31.
- , Review of *A Budget of Paradoxes*, *Bulletin of the American Mathematical Society*, 22 (1916), 468–71.
- Macfarlane, Alexander, *Lectures on Ten British Mathematicians of the Nineteenth Century* (New York: Wiley, 1916).
- Secord, James A., *Victorian Sensation: The Extraordinary Publication, Reception, and Secret Authorship of Vestiges of the Natural History of Creation* (Chicago: University of Chicago Press, 2003). <https://doi.org/10.7208/chicago/9780226158259.001.0001>
- Smith, David Eugene, *History of Mathematics*, 2 vols (Boston: Ginn, 1923).
- Smith, James, [Advertisement] 'The Quadrature of the Circle. To the Editor of the Athenæum', *The Athenæum* (25 May 1861), 679.
- , *The Quadrature of the Circle* (Liverpool: Edward Howell, 1865).
- Stewart, Larry, and Paul Weindling, 'Philosophical Threads: Natural Philosophy and Public Experiment Among the Weavers of Spitalfields', *British Journal for the History of Science*, 28 (1995), 37–62. <https://doi.org/10.1017/s0007087400032684>
- Struik, Dirk J., 'A Story Concerning Euler and Diderot', *Isis*, 31 (1940), 431–32.
- Tait, Peter Guthrie, 'De Morgan's *Budget of Paradoxes*', *Nature* (30 Jan. 1873), 239–40.
- Thiébaud, Dieudonné, *Mes souvenirs de vingt ans de séjour à Berlin*, 5 vols. (Paris: Buisson, 1804–05).