
NO PRICES
NO GAMES!
FOUR ECONOMIC MODELS
michael RICHTER
ariel RUBINSTEIN

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0 Introduction

0.1 The Book

We regard Economic Theory as a collection of models, each viewed as a story or a fable rather than as a testable scientific model to be verified or refuted (see Rubinstein (2012)). Models in Economic Theory are "useful" in the same sense that fables are. Perhaps, there is no boy who literally "cried wolf", but we nevertheless tell the story to teach our children about the dangers of exaggeration. Likewise, the fables we tell in Economic Theory are not meant to be "true" but, rather, are intended to draw our attention to some aspect of real economic life. We view the construction and analysis of models in Economic Theory as a cultural endeavour rather than a scientific one.

Almost all models of interaction between agents in current Economic Theory belong to one of two families: Markets or Games. In market models, there are conflicts over limited resources that are resolved through the emergence of prices, which are taken as given by the agents. These prices bring order to the economic chaos by orchestrating the behaviour of selfish agents. In game-theoretical models, each agent (player) chooses a strategy, and an equilibrium is a profile of strategies such that each agent's strategy is individually optimal, given the correct prediction of other players' behavior. In other words, in a market, each agent chooses his best alternative given the prevailing prices, while in a game, each agent chooses his best strategy based on correct forecasts of what other agents intend to do.

While market models dominated Economic Theory for most of the 20$^{\text{th}}$ century, Game Theory subsequently captured the crown. In the last few decades, Economic Theory has seen another change: economic theorists have liberated themselves from the rigid assumption of full rationality in the pursuit of materialistic goals. The Bounded Rationality literature replaced the

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rationality assumption with explicit reference to decision procedures, while the Behavioral Economics literature added realistic psychological motives to purely materialistic considerations. However, these developments left in place the standard view of economic interactions as being resolved through prices or games.

This short book is aimed primarily at young economists. It is intended to demonstrate models of interaction between agents with NO PRICES and NO GAMES. We do not claim that these models are any more (or less) "true", "realistic", or "useful" than others. In fact, we do not believe that these adjectives are even relevant to models in Economic Theory. As mentioned, we view these models as economic stories: they are interesting; they capture some aspect of reality; they are elegant; they are novel; or ... not.

In the models we study, agents are purely self-interested, and equilibrium reflects a social institution that systematically alters either the agents' choice sets or their preferences. In this respect, the models are closer to market models than to game-theoretical models, and, as in the case of market models, an equilibrium will not be just a profile of choices made by individuals, but will also specify an additional price-like element that uniformly affects all agents.

While we do not have any applied message, working on these topics has brought us to the realization that economic harmony can be achieved by institutions other than prices or games. Of course, this realization could have happened even without any models, but they illuminate how such institutions may function in bringing harmony to economic situations. We focus on four institutions: *Power* (Chapter 1), *Social Norms governing what is permissible and what is forbidden* (Chapter 2), *Status* (Chapter 3), and *Preference Biases* (Chapter 4). In the last part of the book (Chapter 5), we compare our approach to other more established ones. We refrain from any normative assessments of the institutions. Such judgements are left to the reader.

0.2 The Notion of an Economy

The stage on which this book's plots will be performed is a formal model called an *economy*. The model is intended to abstractly capture situations in which each agent in a society chooses an alternative and there exists a fundamental tension between the agents' personal desires and society-wide feasibility constraints. (For example, in a Walrasian economy, consumers have unquenchable desires, but overall resources are limited.) The model's abstraction allows us to consider examples that are "economics" in the conventional sense of the term, but also others that are not. Nevertheless, the term "economy" will be used throughout since all of the models feature the fundamental economic conundrum: individuals' desires cannot all be satisfied due to feasibility constraints on the profiles of choices that can be made in the society.

Definition: Economy

An **economy** is a tuple $\langle N, X, (\succsim^i)_{i\in N}, F\rangle$ where:

- $N = \{1,\ldots,n\}$ is the set of agents.
- X is a set of personal alternatives.
  Each agent chooses an *element* from X. In its most general form, no structure is imposed on X; however, we sometimes consider the special case where X is a subset of a Euclidean space.
- $\succsim^i$ is agent i's preference relation over the set X.
  The fact that preferences are defined over X rather than over the set of choice profiles embodies the assumption that there are no externalities: each agent cares *only* about his chosen alternative irrespective of what other agents choose (as in the case of markets, but unlike in the case of games).

- $F \subset X^N$ is a non-empty set of feasible profiles.
  A choice profile $(x^i)_{i\in N}$ specifies an element $x^i \in X$ for each agent $i \in N$. The set X^N is comprised of all choice profiles. Not all profiles are feasible, and the feasibility constraint is given by a set $F \subset X^N$. Unless stated otherwise, we assume that F is closed under all permutations (i.e. the feasibility constraint is anonymous and does not discriminate between agents). We usually abbreviate $(x^i)_{i\in N}$ as (x^i).

An economy without preferences, $\langle N, X, F\rangle$, is called an **environment**.

Sometimes, we consider an extended version of an economy which specifies for each agent i an element e^i in X, with the interpretation that i always has the right to choose e^i. The vector (e^i) is required to be in F, namely the allocation of these initial rights is feasible. The role of the vector (e^i) is analogous to that of the profile of initial endowments in the standard exchange economy.

Definition: Extended Economy

An **extended economy** is a tuple $\langle N, X, (\succsim^i)_{i\in N}, F, (e^i)_{i\in N}\rangle$ where:

- $\langle N, X, (\succsim^i)_{i\in N}, F\rangle$ is an economy.
- $(e^i)_{i\in N}$ is a feasible initial profile.

0.3 Examples of Economies

We now introduce some economies which appear throughout the book. As mentioned, some of the examples are traditional economic settings while others demonstrate the framework's ability to model a variety of alternative social situations.

Example: The Housing Economy

The set X contains n distinct elements called *houses* (recall that n is the number of agents) and each agent i has preferences $\succsim^i$ over the houses. Each agent chooses a house, but no two agents can occupy the same one. That is, F is the set of profiles that assigns a distinct house to every agent. This economy is the iconic model of Shapley and Scarf (1974). The model is attractive due to its simplicity and its usefulness as a platform for introducing a rich variety of concepts.

If each agent's ideal is distinct, then the situation is "bliss", there are no conflicting desires, and so there is no need for a social institution to achieve harmony in the society. However, bliss does not usually exist, and, therefore, we need social institutions to resolve the conflict between agents' desires and societal feasibility.

Example: The Division Economy

There are K commodities, and the set of alternatives $X = \mathbb{R}^K_+$ consists of the non-negative bundles of those commodities. Preference relations are monotonic, continuous, and convex. As in standard market settings, there are limited resources, and the set of feasible profiles $F = \{(x^i) \mid \Sigma_i x^i = e\}$ is the set of all partitions of a total endowment $e \in \mathbb{R}^K_+$ among the agents. If we would add initial endowments to the model, then we would obtain the classical framework used by economists since Edgeworth (1881) to discuss voluntary exchange and competitive equilibrium. Bliss is always impossible, unlimited wants must be constrained in the face of limited resources, and achieving social harmony requires some social institution.

Example: The Give-and-Take Economy

There are situations in life in which redistribution is imposed by an authority that forces individuals to comply, and there are others in which redistribution is accomplished by means of voluntary exchange between individuals. There are further situations (e.g. a soup kitchen) in which exchange is carried out by unilateral actions: some individuals give while others take without any exercise of power, commitments to "return the favour", or coercion by an authority. These actions are self-motivated: some people like to give, while others like to take. But typically, such motives will not balance each other out, and social norms are needed to achieve harmony.

Formally, we consider the following give-and-take economy, which was first studied by Sprumont (1991). Let $X = [-1,1]$, where a positive x represents a withdrawal of x from a social fund (i.e. taking) and a negative x represents a contribution of $|x|$ to the social fund (i.e. giving). Preferences are assumed to be continuous and strictly convex (that is, single-peaked) but need not be monotonic. Feasibility requires that the social fund is balanced, that is, $F = \{(x^i) \mid \Sigma_i x^i = 0\}$.

Example: The Clubs Economy

The set X consists of a finite set of clubs (see Buchanan (1965)). Each agent chooses a single club to become a member of. Agents have preferences over the clubs and not over the clubs' members. The feasibility constraint is defined by the limits on how many people can belong to each club. Specifically, there is a vector of positive integers $(q_x)_{x\in X}$ where q_x is the quota for club x (for non-triviality, we require that the sum of the quotas is at least n). The set of feasible profiles are those for which no club is chosen by more people than allowed by its capacity.

Example: The Stay Close Economy

This example illustrates the potential of our abstract concept to expand the scope of classical economic analysis. It does not involve goods but nonetheless fits squarely into our concept of an economy. In this example, X is a set of locations in some geographical area. Each agent chooses a location in X and has preferences over the locations. Not every profile of locations is feasible because the society is under threat and its survival depends upon the ability of its members to quickly reach one another in the case of danger. Therefore, all members need to live close enough to each other so that whenever one of them is attacked the others can quickly come to his defence. Formally, the feasibility constraint F requires that the distance between any two agents does not exceed some constant d. When d is very large, every agent can choose his ideal location, but when d is small, this is no longer feasible.

We refer to the special case when $d = 0$ as the *consensus economy*. This fits, for example, the situation of a political party whose members need to present a united front. That is, in order to maintain cohesion, all members of the party need to express the same position.

Example: The Matching Economy

Matching problems are classics of Cooperative Game Theory. Agents have to find a match, and each agent has a preference relation over his potential partners. This situation fits our framework by letting the set of alternatives X be the set of agents N. That is, each agent chooses a partner, which can be himself. Each has a preference relation on X that places himself at the bottom. The feasibility constraint F stipulates that for any i and j, if i chooses j, then j must choose i. Note that this feasibility constraint differs from those in the previous examples in that F is not closed under all permutations.

Example: The Sequential Production Economy

A group of n agents works in n shifts to transform an initial product x^0 into a different product. Each works one shift, and the agents may work in any order. An agent's ability to produce a product, which might be just an intermediate product, depends on the output of the previous shift. The group possesses a technology that enables certain transformations of one product into another.

More precisely, X is a set of products that includes x^0. Each agent has preferences for the product that he produces (rather than for the final product). The common production technology is a correspondence T from X to X where $T(x)$ is the set of outputs which x can be transformed into. Any agent can choose to be "idle" and not transform the product produced in the previous shift, that is $x \in T(x)$. Thus, F is the set of all permutations of profiles $(x^1,\ldots,x^n)$ such that $x^m \in T(x^{m-1})$ for $m = 1,\ldots,n$.

0.4 Equilibrium Concepts

This book introduces and analyzes several solution concepts and applies them to a variety of economic environments. In general, a solution concept relates to some domain of economic environments and determines for each environment a set of harmonious outcomes. These outcomes are harmonious in the sense that the assumed forces that may disturb harmony are neutralized. In our setting, the domain of a solution concept is a class of economies and a candidate for equilibrium typically includes two components:

(i) **A profile of choices** — one choice for each agent.

(ii) A specification of certain **parameters** that systematically influence either agents' choice problems or their preference relations.

Harmony is achieved in equilibrium as follows: agents make individually optimal choices, and the parameters restrict their choice sets (or, in one case,

biases their preferences) to be compatible in the sense that the resulting profile of choices is feasible. The concepts will differ in the parameters and in how they restrict agents' choice sets.

The solution concepts discussed in the book can be divided into two groups. In the **choice group**, each agent's choice set depends on a price-like equilibrium parameter but not on the equilibrium profile of choices. Such choices must be individually optimal and compatible. These concepts are similar in structure to the notion of competitive equilibrium whose parameters are prices and each agent's choice set (budget set) is determined solely by his initial endowment and the prices.

Three of our solution concepts belong to this group:

Y-equilibrium (Chapter 2). The price-like parameter in a Y-equilibrium is a set of alternatives which is interpreted as the set of "permissible" alternatives that uniformly binds all agents. When making a choice, an agent only needs to know the set of permissible alternatives and nothing else. In equilibrium, the permissible set is a maximal set of alternatives from among those which satisfy the following property: if every agent chooses a preference-maximizing alternative from this set, then the resulting choice profile is feasible.

Initial Status Equilibrium (Chapter 3). This concept relates to an extended economy wherein the notion of an economy is enriched with an additional element: a feasible profile of alternatives, one for each agent, in which the alternative designated to an agent is interpreted as one that he always has the right to choose. The price-like parameter in an initial status equilibrium is an ordering of the alternatives that can be interpreted as "status" or "value". An agent's choice set is comprised of all alternatives which have a weakly lower status than his endowment. In equilibrium, a status ordering prevails such that each agent's designated alternative is his most preferred from among his choice set, namely the set of all alternatives that are of weakly lower status than his initial alternative. As always, an equilibrium profile of choices has to be feasible.

Biased Preferences Equilibrium (Chapter 4). The price-like parameter in a biased preferences equilibrium is a vector that systematically biases agents' preferences. In this model, agents' choice sets are fixed and unaffected by the parameters. Rather, in an equilibrium, a systematic bias prevails such that each agent chooses a most-preferred alternative from his choice set, according to his biased preferences, and the profile of choices is feasible.

In the **deviation group** of solution concepts, an equilibrium is a profile of choices that is immune to any single agent's deviation from his prescribed alternative to any alternative in a set determined by the equilibrium parameters. This is the approach taken in Game Theory. For example, a Nash equilibrium is a profile of actions such that, for each agent, the outcome of that profile is not worse for him than any other outcome he can achieve given the other players' choices in the profile.

Two of our solution concepts fall into this group:

Jungle Equilibrium (Chapter 1). In this case, the economy is extended with an exogenous power ranking of the agents; but, in an equilibrium, there are no additional parameters. In the jungle, an agent can steal from those that are weaker than himself; therefore, his choice set is determined by his equilibrium choice as well as the choices of those who are weaker than him. A jungle equilibrium is a profile of choices such that each agent's assigned choice is preference-maximal from among the set of the alternatives he can obtain by stealing resources from weaker agents.

Status Equilibrium (Chapter 3). Again, the price-like equilibrium parameter is an ordering over the alternatives that connotes status (or value). However, in this case, an agent's choice set depends not only on this parameter but also on his own equilibrium alternative. In detail, his choice set is the set of all alternatives which are weakly lower-ranked than his *equilibrium alternative* (rather than his initial alternative). An equilibrium is a status ordering and a profile of optimal choices such that the profile of choices is feasible.

The book analyzes each of these solution concepts both in the abstract, by means of general propositions, and more concretely, by applying the solution concepts to a variety of economic environments (some familiar and some novel).