BEYOND POPULAR SCIENCE

DAVID H. SILVER

BEYOND POPULAR SCIENCE
David H. Silver

https://www.openbookpublishers.com

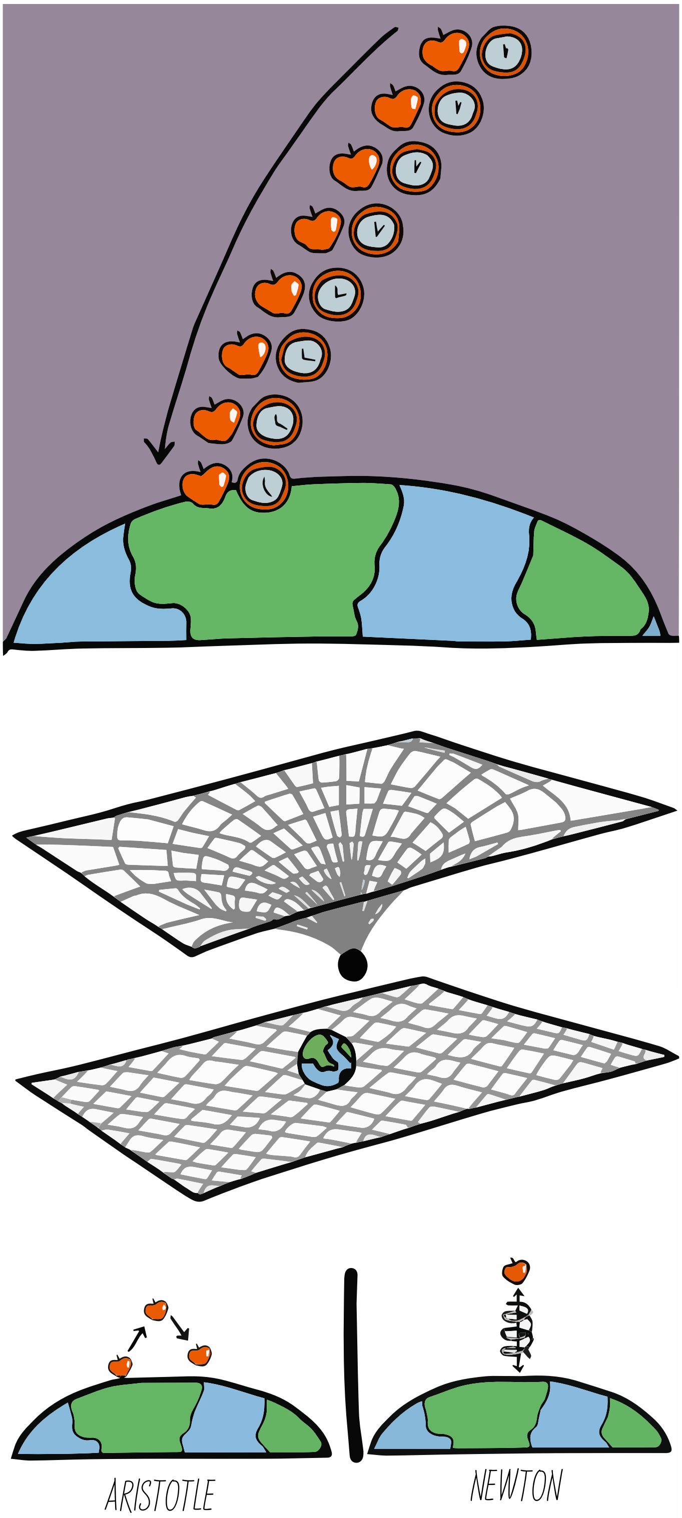
> David H. Silver, *Beyond Popular Science*. Cambridge, UK: Open Book Publishers, 2026, `https://doi.org/10.11647/OBP.0526`

Digital material and resources associated with this volume are available at
`https://doi.org/10.11647/OBP.0526#resources`

ISBN Paperback: 978-1-80511-877-0
ISBN Hardback: 978-1-80511-878-7
ISBN Digital (PDF): 978-1-80511-879-4
ISBN HTML: 978-1-80511-881-7
ISBN Digital ebook (epub): 978-1-80511-880-0
DOI: 10.11647/OBP.0526

Cover image by Enny Silver and David H. Silver
Cover design by Jeevanjot Kaur Nagpal

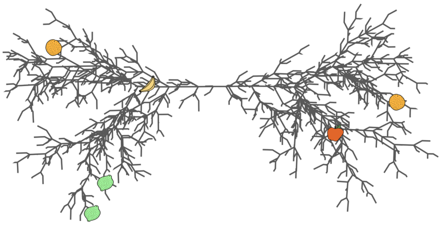
The Apple Falls the Slowest from the Tree



Top (GR—Falling Toward Slow Time): An apple follows a curved free-fall trajectory toward Earth. Along the path, it is shown accompanied by clocks, which tick progressively slower as they get closer to Earth. This is an insight relativity: free-falling objects move toward regions where proper time is maximised—in other words, where the time component of the metric, g_{00}, decreases. The apple is not being pulled by a force, but slides through spacetime toward slower time.
Middle (Misleading Space Bending): Two ‘rubber sheet’ grid diagrams are shown. The top depicts an exaggerated funnel-shaped distortion caused by a black mass—a common pop-science depiction of gravity. The bottom shows the Earth resting on a seemingly flat grid, emphasising that for small masses such as Earth, spatial curvature is negligible. What matters is the distortion of time, not space. Gravity in this regime is governed almost exclusively by the g_{00} gradient.
Bottom (Aristotle to Newton): Left: Aristotle’s model shows apples falling back to Earth because they ‘belong’ there—matter returning to its natural place. Right: Newton’s model depicts a single apple being pulled downward by an invisible gravitational force, with spring lines suggesting mutual attraction between masses. This is Newton’s universal law of gravitation—a force acting at a distance.

The Apple Falls the Slowest from the Tree

General relativity formulates gravity as spacetime curvature where time and space metrics are affected by mass and energy. Contrary to the depiction of gravity as bending space, as in the rubber sheet visualisations, in cases in which masses are small (the Earth, for example) it is the gradient in time's rate that creates gravitational attraction, guiding objects towards regions of slower time. The reason an apple is falling is not because it is affected by force radiated by Earth, nor is it due to the curvature of space, but rather because it is following the shortest path through curved time.



Equivalence Principles ∘ Free Fall as Geodesic ∘ Spacetime Metric $g_{\mu\nu}$ ∘ Gravitational Time Dilation ∘ g_{00} Component ∘ Falling Into Slower Time ∘ Local Inertial Frames ∘ Tidal Effects ∘ Einstein's Elevator ∘ Proper Time Maximisation ∘ GPS Clock Corrections

"Spacetime tells matter how to move;
matter tells spacetime how to curve."
— John Archibald Wheeler, 1962

"We have ways of making people talk...
by giving them fresh apple slices."
— Kenneth Parcell, 2010

The Apple Falls the Slowest from the Tree

Isaac Newton's 1687 *Principia* described gravity as a force acting at a distance between masses, accurately predicting the behaviour of falling objects and planetary orbits. This framework dominated physics for more than two centuries. Yet as astronomical measurements grew more precise, small but persistent anomalies emerged—most notably the unexplained excess precession of Mercury's orbit, which Newtonian mechanics could not account for.

In 1915, Albert Einstein introduced general relativity, reframing gravity as the curvature of spacetime. This insight came from Einstein's realisation that the equivalence of gravitational and inertial mass was no coincidence. His theory predicted phenomena beyond Newton's reach: time would flow differently in gravitational fields, and light would bend twice as much as predicted by Newton when passing massive bodies.

The name ‘general’ relativity captures the theory's ambitious scope—it applies to all observers, whether accelerating, rotating, or in gravitational fields, generalising Einstein's 1905 special relativity which was limited to inertial frames.

Experimental confirmation came in 1919, with Arthur Eddington's expedition to observe a solar eclipse. They found starlight bending around the Sun consistent with Einstein's prediction within experimental uncertainties—instantly making the German physicist a global celebrity. Later experiments provided increasingly precise validation: the 1959 Pound–Rebka experiment detected gravitational redshift using gamma rays in a Harvard tower; Gravity Probe A (1976) launched a hydrogen maser clock on a rocket to confirm time dilation with altitude; and by 1980, ground-based cesium clocks could measure these effects with exquisite precision.

By the late twentieth century, relativistic effects had become engineering concerns—GPS satellites must continuously correct for both special and general relativistic temporal effects to maintain metre-level positioning accuracy.

The ultimate confirmation came a century after Einstein's publication. In 2015, LIGO detected gravitational waves from two black holes spiralling together over a billion light-years away. This observation, followed by dozens more including neutron star collisions, validated Einstein's theory in extreme gravitational regimes.

General relativity remains one of physics's most tested theories, validated from subatomic to cosmological scales.

The weak equivalence principle states that all objects follow identical free-fall trajectories when released from the same point—whether lead or feathers. Formulated by Galileo and tested for centuries, it shows gravitational acceleration is and tested for centuries, it shows gravitational acceleration is independent of mass, charge, or composition. This universality implies that gravitational (how strongly gravity pulls on an object) and inertial mass (the object’s resistance to acceleration) are proportional—a coincidence in Newtonian physics that inspired Einstein to reinterpret gravity geometrically (Einstein, 1915), as motion through curved spacetime.

https://doi.org/10.11647/OBP.0526.06

The strong equivalence principle generalises this: in any local region of spacetime, gravitational effects can be eliminated by choosing a freely falling reference frame. Within such a frame, spacetime curvature becomes negligible. The geometry flattens to first order, and all physical processes proceed as they would in the absence of gravitation. Mechanical systems evolve according to Newton's laws, electromagnetic fields obey Maxwell's equations in vacuum (Maxwell, 1865), and the motion of particles is governed by the inertial character of special relativity.

While spacetime may be globally curved, it admits neighbourhoods indistinguishable from flat Minkowski space. No experiment in a small free-falling laboratory can detect gravity.

Einstein's elevator thought experiment illustrates this: in a free-falling elevator, released objects float weightlessly and light travels straight. No experiment inside can detect the external gravitational field. Mechanical pendulums, electromagnetic resonators, and radioactive decay rates all behave as in gravity-free space. Free fall and inertial motion are locally identical when tidal effects are negligible—that is, when the variation in gravitational field strength across the laboratory is too small to measure. Over larger regions, these tidal effects become detectable as objects at different positions experience slightly different accelerations, causing initially parallel trajectories to converge or diverge.

The spacetime metric is the mathematical object that defines how distances and time intervals are measured. It quantifies the separation between nearby events.

In flat spacetime, the metric is constant (Minkowski, 1908): $ds^2 = -c^2dt^2+dx^2+dy^2+dz^2$. Time gets a negative coefficient $-c^2$ while space coordinates get positive coefficients, meaning the shortest distance between two points is when space separation is minimised and time separation is maximised. In curved spacetime where gravity is present, the metric tensor $g_{\mu\nu}$ varies from point to point. This variation determines how clocks tick at different locations and how distances are measured—it is what we experience as gravity.

These variations manifest in observable ways. Clocks at different gravitational potentials accumulate time at different rates. Initially parallel free-falling trajectories converge or diverge. Tidal effects—the differential forces that stretch objects toward massive bodies and compress them perpendicular to that direction—arise because gravitational field strength varies with position. On Earth, these variations cause ocean tides as the Moon pulls more strongly on the near side than the far side. In extreme cases near black holes, tidal forces can tear objects apart or spaghettify them. When exchanging signals between different altitudes, identical clocks emit pulses at regular intervals but receivers measure changed intervals: upward light is redshifted, downward light is blueshifted.

A local inertial frame has no proper acceleration and follows special-relativistic laws—light travels straight and clocks tick uniformly. The metric matches flat spacetime at a point, with deviations appearing only at second order in displacement. But when comparing such frames at different locations, clocks at different altitudes tick at different rates, transported vectors fail to align, and no single coordinate system makes the metric flat everywhere. The curvature is part of the metric's second derivatives.

A helpful analogy comes from curved surfaces. Imagine two travellers walking north from the equator along different lines of longitude. Their paths begin parallel and appear straight locally, yet they eventually converge at the pole. The meeting point is not due to any force between them but to the geometry. In spacetime, freely falling objects can similarly start with zero relative velocity and later converge or diverge, not because of an interaction, but because the metric changes from point to point.

The metric tensor contains ten independent components in four dimensions. Each component encodes different aspects of spacetime geometry. The spatial components g_{ij} govern how distances are measured and produce effects such as gravitational lensing—light bending around massive objects. The time-space cross terms g_{0i} appear when spacetime itself rotates, as around spinning black holes, causing frame dragging (Lense & Thirring, 1918). But near Earth's surface, where rotation is negligible and velocities remain small compared to light, one component dominates all others in determining motion: g_{00}.

The metric component g_{00} encodes how proper time flows for stationary observers. When g_{00} varies with position, identical coordinate intervals correspond to different amounts of proper time—this is gravitational time dilation. Atomic clocks aboard aeroplanes, satellites, and mountaintops have confirmed these predictions with precision essential for GPS accuracy. While spatial curvature produces phenomena such as gravitational lensing and tidal forces, the gradient of g_{00} determines both the rate at which clocks tick and the direction objects fall.

This connection between time and motion is the essence of gravity. In general relativity, freely falling objects follow geodesics—paths that extremize proper time through spacetime. The insight lies in the metric signature: time enters with an opposite sign to the spatial components $ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$. This opposite sign means that minimising spatial separation and maximising temporal duration both contribute to extremizing the spacetime interval in the same direction. A straight line in space is the shortest spatial path; a geodesic in spacetime is the longest proper time. Near Earth's surface, these geodesics curve downward in space precisely because proper time accumulates more slowly at lower altitudes—moving toward slower clocks maximises the proper time experienced along the path. The 'force' we attribute to gravity is actually the spatial projection of motion along these curved spacetime paths.

Consider an apple hanging from a tree. While attached to the branch, it resists its natural geodesic. Once the stem breaks, the apple enters free fall. As it descends, the changing gradient of g_{00} continuously adjusts its trajectory toward regions where time passes more slowly. This curvature in the apple's spacetime path manifests as what we perceive as gravitational acceleration. The metric itself, encoding how time flows differently at each point in space, tells matter how to move.

Falling Into Slower Time

The realisation that gravity arises from differences in the rate of time's passage, rather than from any applied force, marks a sharp shift in our description of nature. This perspective is mathematically well-defined and supported by high-precision experiments, but it remains difficult to visualise and understand intuitively.

The rubber sheet analogy shows mass distorting a surface, with objects curving toward the indentation. While this conveys how mass alters geometry, it emphasises spatial curvature. In general relativity, gravitational motion comes primarily from varying proper time—time flows more slowly deeper in gravitational fields. This temporal gradient, not spatial curvature, governs free fall.

This is, for me, a *fantastic* observation. An apple falls from a tree not because it is pulled downward, but because time flows slightly faster at the top of the tree than at the bottom. Once released, the apple is no longer constrained. It follows a trajectory through spacetime that maximises the amount of proper time experienced along the way. The curve of this path is determined by how the clock rate changes with altitude.

In everyday reasoning, we often think of objects as taking the shortest route through space. But in general relativity, freely falling objects follow the most direct path through spacetime as a whole, which is about minimising spatial distance while maximising proper time, until even the distinction between space and time becomes irrelevant (as in the vicinity of a black hole which we will discuss in Chapter 23).

A race against time.

Exercise: Deriving the Schwarzschild Metric

Objective: Derive the Schwarzschild metric, which describes spacetime outside a spherically symmetric, non-rotating, uncharged mass M.
Assumptions: Spherical symmetry, static spacetime, vacuum solution ($R_{\mu\nu} = 0$), and the Newtonian limit at large r.

1. **General Form of the Metric.** A spherically symmetric, static spacetime is described by the most general metric:

   $$ds^2 = -A(r)c^2dt^2 + B(r)dr^2 + r^2 d\Omega^2,$$

   where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and $A(r)$, $B(r)$ are functions of r only. *(Why only r? Symmetry! Time-independence and spherical symmetry ensure that metric components cannot depend on t, θ, or ϕ).*
2. **Newtonian Limit.** In the weak-field, slow-motion limit, the metric must reduce to the Newtonian potential: $g_{00} \approx -(1 + 2\Phi/c^2)$, where $\Phi = -GM/r$ is the Newtonian potential of a point mass M. This implies $A(r) = 1 - 2GM/rc^2$. Define the **Schwarzschild radius** $r_s = 2GM/c^2$, so $A(r) = 1 - r_s/r$.
3. **Key Assumption: Relationship Between $A(r)$ and $B(r)$.** Instead of solving Einstein's field equations explicitly, we assume that $B(r) = 1/A(r)$. *Why?*
   a) **Birkhoff's Theorem:** The unique spherically symmetric vacuum solution must be static and match the Schwarzschild metric.
   b) **Energy Conservation in Radial Free-Fall:** If an object falls radially inward, the proper time and coordinate time should be related in a way that matches Newtonian conservation of energy in the weak-field limit.

   This assumption gives:

   $$B(r) = \frac{1}{1 - r_s/r}.$$
4. **The Schwarzschild Metric.** Substituting $A(r) = 1 - r_s/r$ and $B(r) = (1 - r_s/r)^{-1}$ into the general metric:

   $$ds^2 = -\left(1 - r_s/r\right)c^2dt^2 + \frac{dr^2}{1 - r_s/r} + r^2 d\Omega^2.$$
5. **Physical Consequences:**
   a) **Event Horizon ($r = r_s$):** The metric component g_{00} vanishes and g_{rr} diverges. This is a coordinate singularity, marking the event horizon of a black hole.
   b) **Inside the Schwarzschild Radius ($r < r_s$):** The roles of space and time effectively switch, leading to an inevitable collapse toward $r = 0$.
   c) **Observational Predictions:** The Schwarzschild metric predicts gravitational time dilation, light bending, and the precession of planetary orbits (as seen in Mercury).

Gravity as Curved Time: The Time-Only Ansatz

General relativity describes gravity through the curvature of spacetime. But which part of spacetime does the work? Rather than beginning with the full Schwarzschild solution, we follow a more direct route: assume space is perfectly flat and ask whether variations in the rate of time alone can reproduce Newtonian gravity. The answer is yes—and the derivation shows why spatial curvature is irrelevant for everyday gravitational phenomena.

The Ansatz: Flat Space, Variable Time

We assume Euclidean spatial geometry but allow the rate at which clocks tick to depend on position. This gives a spacetime interval of the form:

$$ds^2 = -f(\mathbf{x})\, c^2\, dt^2 + dx^2 + dy^2 + dz^2,$$

where $f(\mathbf{x})$ is a static function encoding how fast a clock runs at location $\mathbf{x}$. In the absence of any gravitational source, $f = 1$ everywhere and we recover the flat Minkowski metric of special relativity.

Connecting f to the Newtonian Potential

The Newtonian gravitational potential outside a mass M is $\Phi(\mathbf{x}) = -GM/r$, where $r = |\mathbf{x}|$. We set:

$$f(\mathbf{x}) = 1 + \frac{2\Phi(\mathbf{x})}{c^2}.$$

Since $\Phi < 0$ near a mass, we have $f < 1$: clocks deeper in the gravitational well tick slower. The dimensionless ratio $|\Phi|/c^2$ measures the strength of the effect. At Earth's surface, $GM/(c^2 R_\oplus) \approx 7 \times 10^{-10}$—a tiny perturbation, but one with real consequences.

Proper Time and the Geodesic Equation

A freely falling particle follows a geodesic—the path that maximises proper time between two events. The proper time along a worldline is:

$$d\tau^2 = f(\mathbf{x})\, dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2).$$

For a particle moving slowly ($v \ll c$), the spatial terms are negligible compared to the time term, and $d\tau \approx \sqrt{f}\, dt$. The particle's four-velocity is dominated by $u^0 \approx c/\sqrt{f}$, with spatial components $u^i \ll u^0$.

The geodesic equation $\ddot{x}^\mu + \Gamma^\mu_{\alpha\beta}\, \dot{x}^\alpha \dot{x}^\beta = 0$ then simplifies. Because spatial velocities are small, only the Γ^i_{00} Christoffel symbol matters. For our diagonal metric with flat spatial part $\Gamma^i_{00} = -\frac{1}{2}\, \delta^{ij}\, \frac{\partial f}{\partial x^j} = -\frac{1}{c^2}\, \frac{\partial \Phi}{\partial x^i}$.

Recovering Newton's Second Law

Substituting into the spatial geodesic equation gives the coordinate acceleration:

$$\frac{d^2 x^i}{dt^2} \approx -c^2\, \Gamma^i_{00} = -\frac{\partial \Phi}{\partial x^i}.$$

In vector form: $\mathbf{a} = -\nabla\Phi$. For a point mass, $\Phi = -GM/r$ yields $\mathbf{a} = -GM\hat{\mathbf{r}}/r^2$, which is Newton's inverse-square law. No spatial curvature was needed at any step. The gravitational acceleration arises entirely from the spatial gradient of the clock-rate function f.

How Small Is the Spatial Curvature Correction?

In the full Schwarzschild solution, the spatial metric component $g_{rr} = (1 - 2GM/c^2 r)^{-1}$ also deviates from unity. This introduces an additional Christoffel symbol $\Gamma^r_{rr} \approx -GM/(c^2 r^2)$, which contributes to the geodesic equation via the term $-\Gamma^r_{rr}(v^r)^2$. For an object falling from rest near Earth's surface, $v^2 \sim 2GM/R_\oplus$, so the spatial curvature contribution to acceleration is of order:

$$\frac{a_{\text{spatial}}}{a_{\text{temporal}}} \sim \frac{v^2}{c^2} \approx 2\frac{GM}{c^2 R_\oplus} \approx 1.4 \times 10^{-9}.$$

The spatial geometry of general relativity contributes less than two parts per billion to the gravitational acceleration of a falling apple. For all non-relativistic experience, gravity is an effect of temporal curvature alone.

References:

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation.* (Section 17.4: Newton's Theory in the Language of Spacetime Curvature).

I thank Prof. A. Winkler for suggesting this presentation.