BEYOND POPULAR SCIENCE

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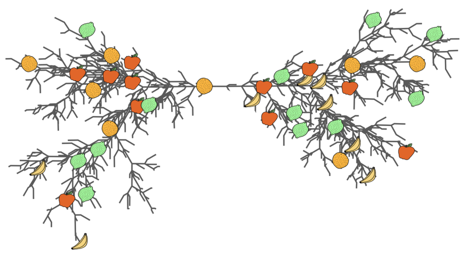
Superanony-
mous

Superpermutation over Anime Orders: The sequence 1,2,3,1,2,1,3,2,1 visits all six permutations of $\{1, 2, 3\}$ as contiguous subsequences: 123, 231, 312, 213, 132, 321. Here, episode '1' is *Dragon Ball Z* (Vegeta), '2' is *One Piece* (Straw Hat Crew), and '3' is *Sailor Moon*. This is a minimal superpermutation for $n = 3$: the shortest possible viewing order that guarantees every order of watching (back-to-back) all three shows appears.

Superanonymous

A significant combinatorial breakthrough from an unlikely source: an anonymous 4chan post (2011) responding to a question about anime episode viewing orders. Superpermutations are strings containing every possible ordering of n symbols as substrings. For years, mathematicians believed the minimal length followed the pattern of factorial sums observed in small cases. The anonymous poster derived a rigorous lower bound, modelling the problem as path optimisation through a permutation graph. This proof remained obscure until 2014 when mathematician Robin Houston rediscovered it, leading to the disproof of the long-standing conjecture and establishing new bounds on this combinatorial problem—with the original derivation still officially credited to 'Anonymous 4chan Poster.'



Superpermutation Problem ∘ Permutation Overlap ∘ Hamiltonian Path Approach ∘ Sum-of-Factorials Conjecture ∘ n=6 Counterexample ∘ Anonymous 4chan Proof ∘ Haruhi Anime Origin ∘ Robin Houston Discovery ∘ Greg Egan Upper Bound ∘ Combinatorial Compression ∘ Non-Academic Mathematics

"Why should I refuse a good dinner
simply because I don't understand the digestive processes involved."
— Oliver Heaviside, 1893

Superanonymous

> From the late nineteenth century, combinatorial mathematics developed tools for enumerating and arranging discrete structures. Permutations—ordered arrangements of elements—became central objects of study, with applications ranging from algebra to scheduling theory. One question was about sequences that embed all permutations of a given set as contiguous substrings. Though such sequences appeared in scattered contexts, the idea of minimising their length—the superpermutation problem—remained informal and largely unexplored.
>
> By the late twentieth century, empirical computer exploration suggested that for small n the minimal lengths matched the sum-of-factorials pattern, $L(n) = \sum_{k=1}^{n} k!$. This led to a widely discussed but unproven conjecture that the pattern might hold in general.
>
> The situation changed dramatically in 2011 when an anonymous user on 4chan's science board posed a variation of the problem in the context of anime episode viewing order. In response, another user posted a rigorous lower bound on superpermutation length, unnoticed by the broader community for years. Independent developments followed: in 2014, a construction of length 872 for $n = 6$ appeared, disproving the factorial-sum conjecture. Soon after, mathematicians formalised the 4chan insight, and Greg Egan proposed a new upper bound, narrowing the known range.

A *permutation* is an ordered arrangement of a set of distinct elements. Consider the set $\{1, 2, 3\}$. One possible ordering is 123, where the elements appear in their natural order. Another is 231, where 2 comes first, followed by 3 and 1. Each such arrangement—where all elements are used exactly once and appear in a specific sequence—is called a permutation of the set.

For a set of size n, the total number of such arrangements is given by the factorial function: $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. The number of permutations grows rapidly: $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, $7! = 5{,}040$. This rapid—factorial—growth means that although the idea of permutations is elementary, the total number becomes intractable to list or store explicitly even for moderate n.

Suppose one attempts to list all permutations of $\{1, 2, 3\}$: these are 123, 132, 213, 231, 312, and 321. Each permutation has length 3. If one were to list them end-to-end it would yield a string of length $6 \times 3 = 18$. For larger n, such direct listings become prohibitively long.

A string that contains all permutations as contiguous substrings is called a *superpermutation*. For example, the naive concatenation 123132213231312321 is a superpermutation of $\{1, 2, 3\}$ because it contains all six permutations as contiguous substrings. However, it has length 18, which is not the minimal possible length. While the naive approach gives a superpermutation of length $n! * n$, it can be *compressed* and made smaller by reusing overlapping segments wherever possible.

Consider the case $n = 2$. The two permutations of the symbols $\{1, 2\}$ are 12 and 21. A superpermutation must therefore include both 12 and 21 as substrings. The shortest such

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string is 121. It contains 12 starting at position 1 and 21 starting at position 2. This is the minimal superpermutation for $n = 2$, and it has length 3.

For $n = 3$, there are $3! = 6$ permutations: 123, 132, 213, 231, 312, and 321. One example of a minimal superpermutation that includes all six of these as contiguous substrings is 123121321. It has length 9, and each of the six permutations occurs once within it. No shorter string satisfies the same condition.

The central question posed by the *superpermutation problem* is: what is the minimal possible length of such a string for general n? That is, given n distinct symbols, what is the shortest string over those symbols that contains every one of their $n!$ permutations as contiguous substrings? As n increases, this problem becomes computationally and combinatorially challenging. The number of permutations grows rapidly, and so does the complexity of arranging them with maximal overlap. The search for minimal superpermutations remains an active area of combinatorial optimisation.

At first glance, the problem of constructing a superpermutation may appear manageable. One might attempt a straightforward solution by writing out all $n!$ permutations of the n symbols and concatenating them end-to-end. Since each permutation is of length n, this method produces a string of total length $n \cdot n!$. For example, for $n = 3$, this naive approach would yield a string of length $3 \times 6 = 18$. While this guarantees that all permutations are present, it is highly inefficient. Adjacent permutations often share common segments—such as matching suffixes and prefixes—and these overlaps can be exploited to significantly reduce the total length.

The trick is that permutations can be arranged so that the end of one serves as the beginning of the next. For instance, the permutation 123 ends in 23, and 231 begins with 23; by placing them consecutively as 1231, the two permutations are both represented, and the shared segment 23 is not duplicated. This principle of overlap allows one to compress multiple permutations into a single string, hopefully, without repeating identical sequences.

For small values of n, exact solutions have been found. Remarkably, for $n = 1$ through $n = 5$, the shortest superpermutations are known, and their lengths follow a simple closed-form pattern:

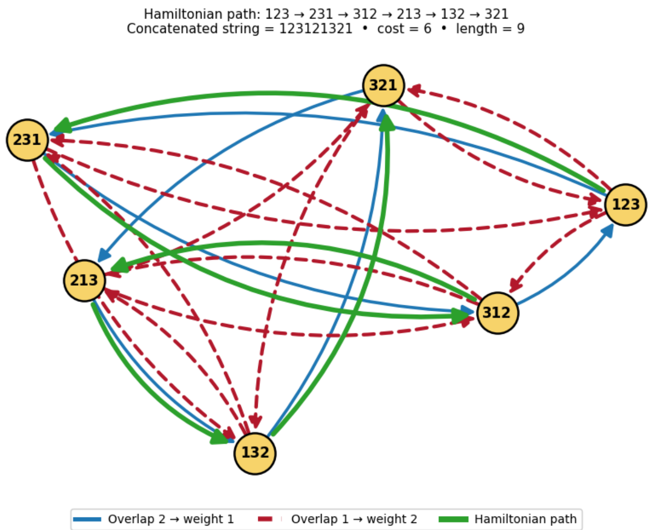
$$L(n) = \sum_{k=1}^{n} k! = 1! + 2! + 3! + \cdots + n!.$$

For example:

$$L(3) = 1! + 2! + 3! = 1 + 2 + 6 = 9, \quad L(4) = 33, \quad L(5) = 153.$$

This empirical closed-form suggested a natural conjecture: that the shortest possible superpermutation on n symbols always has length equal to $\sum_{k=1}^{n} k!$. The conjecture was elegant and aligned with all known cases. For years, no counterexamples were found, and the formula became widely assumed to be correct.

That assumption remained unchallenged until 2014, when it was definitively disproven. A construction was found that produced a superpermutation on six symbols with total length 872—precisely one character shorter than the conjectured value of $1!+2!+3!+4!+5!+6! =$



Permutation overlap digraph for $n = 3$. Each node is a permutation of $\{1, 2, 3\}$. A directed edge $A \to B$ exists when a suffix of A matches a prefix of B, with *overlap* length k. The *weight* is $w = n - k$, i.e. the number of extra symbols needed to append B after A. Edges with $k = 2$ (weight 1) are shown in solid blue, edges with $k = 1$ (weight 2) in dashed red. The highlighted green path is the Hamiltonian path $123 \to 231 \to 312 \to 213 \to 132 \to 321$, yielding concatenated string 123121321 of length 9 and total cost 6.

873. This counterexample showed that the conjectured bound, though valid for $n \leq 5$, does not hold in general.

The origin of this disproof traces back to an unexpected source: an anonymous discussion thread on the imageboard website 4chan. Founded in 2003, 4chan hosts ephemeral user-generated content across numerous boards organised by theme. Messages are anonymous by default, and threads are subject to automatic deletion without archival. The site's culture is informal, transgressive, and often dismissive of academic conventions. One of its boards, labelled `/sci/`, is nominally dedicated to science and mathematics. The board nonetheless occasionally features serious technical inquiry.

In 2011, an anonymous user on the `/sci/` board of the website 4chan posed a question that, at first glance, seemed whimsical: what is the shortest possible viewing sequence that includes every ordering of the 14 episodes of the anime *The Melancholy of Haruhi Suzumiya*? The show, known for its non-linear narrative and varying episode orders across different broadcasts, had developed a cult following that embraced its combinatorial potential. Beneath the framing, however, lay a precise mathematical question: how short can a string be while still containing every permutation of a 14-element set as a contiguous substring? In effect, the prompt was a popular-culture formulation of the superpermutation problem for $n = 14$.

In response, another anonymous poster provided a compact but mathematically rigorous derivation of a new general lower bound (Anonymous et al., 2018): $L(n) \geq n! + (n-1)! + (n-2)! + n - 3$.

This inequality, valid for all $n \geq 2$, strengthened all previously known bounds. Though presented informally, the proof was ultimately correct. It treated permutations as vertices in a directed graph, with directed edges representing overlaps between adjacent substrings. The minimal-length superpermutation corresponded to a Hamiltonian path through this graph (Ashlock & Tillotson, 1993), and the proof established a lower bound on the total cost of any such path by analysing the unavoidable overlaps.

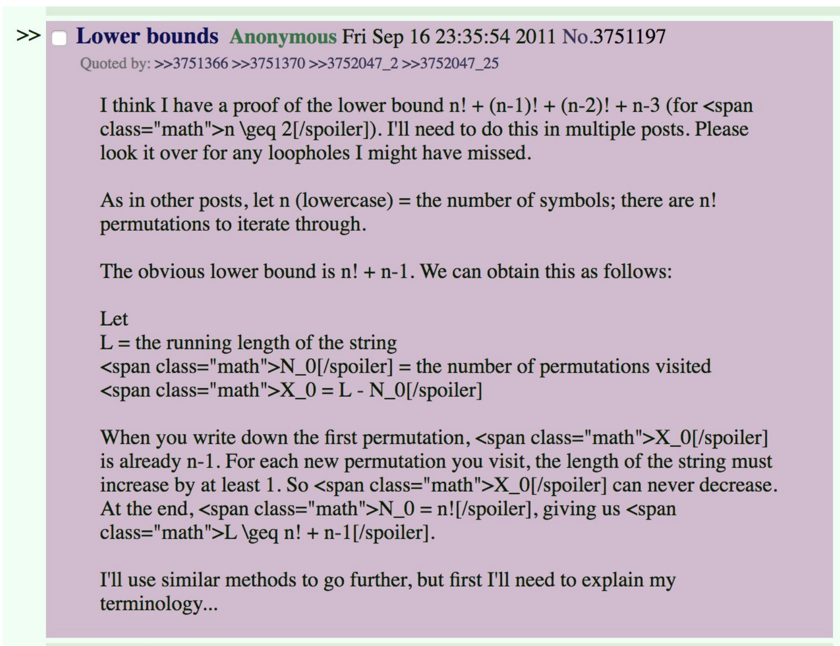
Despite its correctness and novelty, the result went largely unnoticed at the time. The platform offered no mechanisms for citation or persistence: posts were anonymous, threads expired automatically, and archival relied entirely on user initiative. The derivation was eventually copied to a fandom-hosted mathematics wiki, but remained obscure and disconnected from formal literature.

In 2014, mathematician Robin Houston independently rediscovered the argument, verified its correctness, and recognised its significance. He publicised the result, incorporated it into ongoing research, and cited the unknown author as "Anonymous 4chan Poster"—a designation that remains standard in subsequent academic references. The original poster has never been identified.

The impact was immediate. The new lower bound provided a rigorous floor against which all proposed constructions could be measured. Shortly thereafter, Houston constructed a superpermutation on six symbols of length 872 (Houston, 2014)—one less than the conjectured minimum of 873—thereby disproving the long-standing sum-of-factorials conjecture.

In 2018, an accomplished sci-fi author and mathematician Greg Egan proposed a constructive upper bound: $L(n) \leq n! + (n-1)! + (n-2)! + (n-3)! + n - 3$, placing the known range for $L(n)$ within a narrow window, bounded, yet not tightly, from above and from below.

The 4chan derivation now stands as a rare episode in modern mathematics: a significant and previously unknown lower bound for a classical problem, derived anonymously, informally posted, largely ignored, and later validated by professionals. It illustrates how insight can originate outside institutional settings, and how easily such insight can be lost when detached from systems of attribution, preservation, and dissemination. Nevertheless, the mathematics holds. The 'Haruhi Problem' has since entered the literature as a textbook case in combinatorial optimisation—and its solution, at least in part, belongs to a nameless contributor with no affiliation, no traceable authorship, and a correct idea.



>> Lower bounds Anonymous Fri Sep 16 23:35:54 2011 No.3751197
Quoted by: >>3751366 >>3751370 >>3752047_2 >>3752047_25

I think I have a proof of the lower bound n! + (n-1)! + (n-2)! + n-3 (for <span class="math">n \geq 2[/spoiler]). I'll need to do this in multiple posts. Please look it over for any loopholes I might have missed.

As in other posts, let n (lowercase) = the number of symbols; there are n! permutations to iterate through.

The obvious lower bound is n! + n-1. We can obtain this as follows:

Let
L = the running length of the string
<span class="math">N_0[/spoiler] = the number of permutations visited
<span class="math">X_0 = L - N_0[/spoiler]

When you write down the first permutation, <span class="math">X_0[/spoiler] is already n-1. For each new permutation you visit, the length of the string must increase by at least 1. So <span class="math">X_0[/spoiler] can never decrease. At the end, <span class="math">N_0 = n![/spoiler], giving us <span class="math">L \geq n! + n-1[/spoiler].

I'll use similar methods to go further, but first I'll need to explain my terminology...

The original 4chan post

Lower and Upper Bounds via Overlap Graphs

Introduction

A superpermutation on n symbols is a string that contains all $n!$ permutations of those symbols as contiguous substrings. Let $L(n)$ denote the minimum possible length of such a string. The problem of determining $L(n)$ can be reformulated as a path-finding problem on a directed graph whose nodes are permutations and whose edge weights correspond to symbol overlap.

Permutation Graph Model

Let S_n be the symmetric group on n elements, and let each vertex in the directed graph G_n correspond to a permutation $\pi \in S_n$. For each ordered pair (π, σ), define an edge $\pi \to \sigma$ with weight $w(\pi, \sigma) = n - \ell(\pi, \sigma)$, where $\ell(\pi, \sigma)$ is the length of the longest suffix of π that matches a prefix of σ. A superpermutation corresponds to a Hamiltonian path through G_n, with total cost equal to the sum of edge weights plus n.

Anonymous Lower Bound

The anonymous 4chan user showed that for all $n \geq 2$, the minimal length satisfies

$$L(n) \geq n! + (n-1)! + (n-2)! + n - 3.$$

The proof bounds the number of edges in any Hamiltonian path that must have weight greater than 1. Let $\mathcal{P}$ be any Hamiltonian path through G_n. At best, two permutations can overlap by $n-1$ symbols, requiring only one new symbol to transition. However, weight-1 transitions alone cannot form a complete path due to overlap constraints. The proof partitions permutations into blocks where one block must be traversed without overlap (initial permutation); some fraction of transitions must necessarily use edges with higher cost due to incompatible suffix-prefix structure. Using known bounds on minimal-overlap transitions, one can count higher-cost edges and compute their contribution. The derived lower bound is tight for $n \leq 4$ and remains the strongest known general lower bound for $L(n)$.

Egan's Constructive Upper Bound

Greg Egan constructed a general method for building superpermutations of length at most

$$L(n) \leq n! + (n-1)! + (n-2)! + (n-3)! + n - 3.$$

The method generates Hamiltonian paths through subsets of permutations with controlled overlaps: Begin with a path that efficiently traverses permutations of $n-1$ symbols. Lift the path into S_n by inserting the new symbol in controlled positions. Design the insertion and merge process to preserve maximal overlaps where possible. Egan's construction uses Cayley graph traversal techniques and reuses structural symmetries. The difference between the upper and lower bounds is $(n-3)!$:

$$\Delta(n) = L_{\text{upper}}(n) - L_{\text{lower}}(n) = (n-3)!.$$

Example: The Case $n = 7$

Applying the bounds:

$$\begin{aligned} \text{Lower bound:} \quad & 7! + 6! + 5! + 7 - 3 \\ & = 5884, \\ \text{Upper bound:} \quad & \text{lower bound} + 4! \\ & = 5884 + 24 \\ & = 5908. \end{aligned}$$

The shortest known construction is 5906. The true value of $L(n)$ for $n = 7$ remains unknown.

References:

Houston, R. (2014). *Obvious Does Not Imply True: The Minimal Superpermutation Conjecture Is False.* arXiv:1408.5108.
Egan, G. (2018). *Superpermutations* (online note).
Anonymous 4chan Poster (2011). Lower bound proof (`https://warosu.org/sci/thread/3751105`).